Answer on Question #57673, Math/ Analytic Geometry

A circle of radius square root of 10 touches the line x - 3y = 2. Find the general equation of the locus of its center. (Two solutions)

Solution: r= $\sqrt{10}$;O (x,y)-center of the such circle.

The locus of the center such circles is two lines which is parallel to the line x - 3y = 2. The distances between new lines and given line is square root of 10.

The line x - 3y = 2 through the point A(2,0) (for example),

The general equation for the parallel line is x-3y=C; C-some constant, so O belongs to x-3y=C.

The distance from A to the line x-3y=C is r= $\sqrt{10}$, so find C from this condition.

$$\mathrm{distance}(ax+by+c=0,(x_0,y_0))=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}.$$

We get

$$\sqrt{10}=\frac{|2-C|}{\sqrt{1^2+(-3)^3}}=\frac{|2-C|}{\sqrt{10}}$$

|2-C|=10; C_1=12; C_2=-8;

The equation of the locus L_1: x-3y=12 ; L_2: x-3y=-8;

Answer: L_1: x-3y=12 ; L_2: x-3y=-8.

Find the general equations of the bisectors of the angles between the lines:
a. 3x + y = 6 and x + 3y = -2
b. 4x - 5y = 26 and 5x + 4y = 20

Solution:

The equations of the bisectors of the angles between the lines $a_1x+b_1y+c_1$ =0; $a_2x+b_2y+c_2$ =0:

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^{\ 2}+b_1^{\ 2}}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^{\ 2}+b^{\ 2}}}$$

a. 3x + y = 6 and x + 3y = -2;

3x + y – 6=0 and x + 3y +2=0;

$$\frac{3x+y-6}{\sqrt{3^2+1^2}}=\pm\frac{x+3y+2}{\sqrt{1^2+3^2}}\ ;\ \frac{3x+y-6}{\sqrt{10}}=\pm\frac{x+3y+2}{\sqrt{10}}\ ;$$

3x + y – 6= x + 3y +2;

2x-2y-8=0

x-y-4=0;

3x + y – 6= -(x + 3y +2);

3x + y – 6=-x-3y-2;

4x+4y-4=0;

x+y-1=0;

b. 4x - 5y = 26 and 5x + 4y = 20;

4x - 5y -26=0 and 5x + 4y – 20=0;

$$\frac{4x-5y-26}{\sqrt{4^2+(-5)^2}}=\pm\frac{5x+4y-20}{\sqrt{5^2+4^2}};\ \frac{4x-5y-26}{\sqrt{41}}=\pm\frac{5x+4y-20}{\sqrt{41}}$$

4x - 5y -26=5x + 4y – 20;
-x-9y-6=0;
x+9y+6=0;

4x - 5y -26=-(5x + 4y – 20);
4x - 5y -26=-5x-4y+20;
9x-y-46=0;

Answer: a) x-y-4=0; x+y-1=0; b) x+9y+6=0; 9x-y-46=0.

Find the general equation of the line through the point A(-1, 2) and passing at a distance 3 from the point Q(-10, -5).

Solution: The general equation of the line through the point A(-1, 2) : y-2=k(x+1);
-kx+y-k-2=0;

$$\text{distance}(ax+by+c=0,(x_0,y_0))=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}.$$

3= distance(y-kx-k-2; Q(-10,-5))= $\frac{|10k-5-k-2|}{\sqrt{(-k)^2+1^2}}=\frac{|9k-7|}{\sqrt{k^2+1^2}}$;

$\frac{|9k-7|}{\sqrt{k^2+1}}$ =3; squaring the expression: $(9k-7)^2=9(k^2+1)$;
81k^2-126k+49=9k^2+9;
72k^2-126k+40=0;
36k^2-63k+20=0;
D= (-63)^2-4*36*20=3969-2880=1089=33^2;
$k_1=\frac{63+33}{72}=\frac{4}{3}$;
$k_2=\frac{63-33}{72}=\frac{5}{12}$;

L_1: $-\frac{4}{3}x+y-\frac{4}{3}-2=0$; simplifying the equation we get L_1: -4x+3y-10=0;
L_2: $-\frac{5}{12}x+y-\frac{5}{12}-2=0$; simplifying the equation we get L_2: -5x+12y-29=0;

Answer: -4x+3y-10=0; -5x+12y-29=0;

A circle of radius 6 touches both the coordinate axes. A line with slope -3/4 passes over and just touches the circle. If the circle is in the first quadrant, find the general equation of the line.

Solution. The equation of the line with slope -3/4 is y= $-\frac{3}{4}$x+b**;** If the circle is in the first quadrant we get that its center is O(6,6); The distance from O to the line y= $-\frac{3}{4}$x+c equals the radius 6, rewrite y= $-\frac{3}{4}$x+c as 4y+3x-4c=0;

$$\text{distance}(ax+by+c=0,(x_0,y_0))=\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}.$$

6= $\frac{|a\ x_0+b\ y_0+c\ |}{\sqrt{a^{\ 2}+b^{\ 2}}}=\frac{|3*6+4*6-4c|}{\sqrt{3^{\ 2}+4^{\ 2}}}=\frac{|18+24-4c|}{\sqrt{25}}$

6= $\frac{|42-4c|}{5}$; |42-4c|=30; c=3; c=18; L_1: 3x+4y -12=0; L_2: 3x+4y -72=0,

but L_2: 3x+4y -72=0 passes over circle.

Answer: 3x+4y -72=0.