Answer on question №57671, Math / Analytic Geometry

Task:

Write the general equation of the line satisfying the given conditions:

1. passing trough (1, 0) and parallel to the line through (1/2, 3) and (0,0)
2. passing through (2, -4) and parallel to the line 3x - 7y - 11 = 0
3. passing through (-1/3, 1/2) and perpendicular to the line through (4, 1) and (-2, 7)
4. passing through (-1/2, 2) and perpendicular to the line with slope -5/2
5. passing through the point of intersection of lines 5x - 2y - 12 = 0 and x + 3y + 1 = 0 and perpendicular to the line 7x + 4y - 10 = 0.

6.The base of a triangle is the line segment from (3,0) to (2,-3). If the area of the triangle is 7, find the general equation of the locus of the third vertex. (Two solutions).

7. Find the distance between the given lines:

a. 3x - y = 5 and 3x - y = 25

b. 4x - 6y = 9 and 2x - 3y = 6

Solution:

1) Find the line through $(\frac{1}{2}, 3)$ and $(0,0)$. The equation of the line: $y = ax + b$.
   $(0,0)$ is in the line, so 0=a*0+b. then $b = 0$. And $(\frac{1}{2}, 3)$ is in the line, so $3 = \frac{a}{2}$.
   Then $a = 6$. We have a line $y = 6x$. The line parallel to this is $y = 6x + c$. It passing trough $(1, 0)$. So $0 = 6 * 1 + c$. Then $c = -6$.
   Answer: $y = 6x - 6$.
2) The line parallel to $3x - 7y - 11 = 0$ is $3x - 7y + d = 0$. It passing trough $(2, -4)$. So $3 * 2 - 7(-4) + d = 0;\ 6 + 28 + d = 0$. Then $d = -34$.
   Answer: $3x - 7y - 34 = 0$.
3) Like in 1st task find the line through $(4, 1)$ and $(-2,7)$. The equation of the line: $y = ax + b$.
   $$\begin{cases} 1 = a * 4 + b \\ 7 = a * (-2) + b \end{cases}$$
   From the 1st equation $b = 1 - 4a$. Replace to the 2nd equation.
   $7 = -2a + (1 - 4a)$;
   $7 = -6a + 1$;
   $6 = -6a$;
   $a = -1$. And $b = 1 - 4(-1) = 1 + 4 = 5$. So the line $y = -x + 5$. It is equivalent to $+y - 5 = 0$. The line perpendicular to this is $x - y + d = 0$.
   It passing through $(-\frac{1}{3}, \frac{1}{2})$. So $-\frac{1}{3} - \frac{1}{2} + d = 0$. $d = \frac{5}{6}$.
   Answer: $x - y + \frac{5}{6} = 0$.

4) The line with slope $-\frac{5}{2}$ is $y = -\frac{5}{2}x + c$. It is equivalent to $\frac{5}{2}x + y - c = 0$. The line perpendicular to this is $x - \frac{5}{2}y + d = 0$. It passing through $(-\frac{1}{2}, 2)$.
So $-\frac{1}{2} - \frac{5}{2} * 2 + d = 0.\ d = 5 + \frac{1}{2} = \frac{11}{2}$.
Answer: $x - \frac{5}{2}y + \frac{11}{2} = 0$.

5) The point of intersection of lines $5x - 2y - 12 = 0$ and $x + 3y + 1 = 0$ is
$\begin{cases} 5x - 2y - 12 = 0 \\ x + 3y + 1 = 0 \end{cases}$.
Solve this system. From the 2nd equation $x = -3y - 1$. Replace to the 1st equation.
$5(-3y - 1) - 2y - 12 = 0;$
$-15y - 5 - 2y - 12 = 0;$
$-17y - 17 = 0;$
$y = -1$. So $x = -3(-1) - 1 = 2$. So the point of intersection is $(2, -1)$.
The line perpendicular to the line $7x + 4y - 10 = 0$ is $7x + 4y + d = 0$. And through the point $(2, -1)$ is $14 - 4 + d = 0.\ d = -10$.
Answer: $7x + 4y - 10 = 0$.

6) The base of a triangle is the line segment from (3,0) to (2,-3). If the area of the triangle is 7, find the general equation of the locus of the third vertex. (Two solutions).
The line through (3,0) and $(2, -3)$ is $y = ax + b$.

$\begin{cases} 0 = a * 3 + b \\ -3 = a * 2 + b \end{cases}$.

From the 1st equation $b = -3a$. Replace to the 2nd equation.
$-3 = 2a - 3a;$
$-3 = -a;$
$a = 3$. And $b = -3 * 3 = -9$.
So the line is $y = 3x - 9$. It is equivalent to $3x - y - 9 = 0$. (1)
The 3rd vertex is on the line perpendicular to line (1) and through the middle of base.
The middle of the base is the point with coordinates: $= \frac{3+2}{2} = \frac{5}{2}; y = \frac{0-3}{2} = \frac{-3}{2}$.
The line perpendicular to line (1) is $-x + 3y + d = 0$. And through the point $(\frac{5}{2}, -\frac{3}{2})$ is
$-\frac{5}{2} + 3(\frac{-3}{2}) + d = 0;$
$-\frac{5}{2} - \frac{9}{2} + d = 0;$
$d = 7$.
The 3rd vertex is on the line $-x + 3y + 7 = 0$. Denote the vertex of triangle as $(x_1, y_1), (x_2, y_2) and\ (x_3, y_3)$, where $(x_1, y_1) = (3,0) and\ (x_2, y_2) = (2, -3)$. The formula of area of the triangle is
$Area = \frac{1}{2}(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3)$. From the task $Area = 7$.
So $7 = \frac{1}{2}(-9 + 2y_3 + 0 - 0 - x_3(-3) - 3y_3)$.
$14 = -9 - y_3 + 3x_3;$
$3x_3 - y_3 = 23$.
And $-x_3 + 3y_3 + 7 = 0$.

We have a system

$$\begin{cases} 3x_3 - y_3 = 23 \\ -x_3 + 3y_3 + 7 = 0 \end{cases}.$$

From the 2nd equation $x_3 = 3y_3 + 7$. Replace to the 1st equation.

$3(3y_3 + 7) - y_3 = 23;$

$9y_3 + 21 - y_3 = 23;$

$8y_3 = 2.$

$y_3 = \frac{1}{4}. x_3 = \frac{3}{4} + 7 = \frac{31}{4}.$

So the third vertex is $C\left(\frac{31}{4}, \frac{1}{4}\right)$. Another solution is the point $D(x, y)$ in the line is on the line $-x + 3y + 7 = 0$, and symmetric about the middle of base $M(\frac{5}{2}, -\frac{3}{2})$.

From the formula of the middle of line segment : $\frac{5}{2} = \frac{\frac{31}{4}+x}{2}; -\frac{3}{2} = \frac{\frac{1}{4}+y}{2}.$

$5 = \frac{31}{4} + x; -3 = \frac{1}{4} + y.$

$x = -\frac{11}{4}; y = -\frac{13}{4}.$

Answer: $\left(\frac{31}{4}, \frac{1}{4}\right)$ and $\left(-\frac{11}{4}, -\frac{13}{4}\right)$.

7)

When the lines are given by

$$ax + by + c_1 = 0$$
$$ax + by + c_2 = 0,$$

the distance between them can be expressed as

$$d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}.$$

a) Lines $3x - y = 5$ and $3x - y = 25$. The distance is equal $\frac{|5-25|}{\sqrt{3^2+(-1)^2}} = \frac{20}{\sqrt{10}}$

Answer: $\frac{20}{\sqrt{10}}$

b) Lines $4x - 6y = 9$ and $2x - 3y = 6$. The second equation is equivalent to $4x - 6y = 12$. The distance is equal $\frac{|9-12|}{\sqrt{4^2+(-6)^2}} = \frac{3}{\sqrt{16+36}} = \frac{3}{\sqrt{52}}$

Answer: $\frac{3}{\sqrt{52}}$