Answer the question #57669 – Mathematics – Analytic Geometry

Find the angle formed from line 1 to line 2:
a. line 1: $4x - 5y += 0$; line 2: $6x - 4y - 12 = 0$ (*may be a bug!*)
b. line 1: $2x + 7y = 0$; line 2: $3x - 5y - 15 = 0$

Solution.

a. Let α be an angle formed from line 1 to line 2, then (by corresponding formula)

$$\cos\alpha = \frac{4\cdot 6 + (-5)\cdot(-4)}{\sqrt{4^2 + (-5)^2}\cdot\sqrt{6^2 + (-4)^2}} = \frac{24 + 20}{\sqrt{16 + 25}\cdot\sqrt{36 + 16}} = \frac{44}{\sqrt{41\cdot 52}} = \frac{22}{\sqrt{533}}$$

and $\alpha = \arccos\left(\frac{22}{\sqrt{533}}\right) \approx 17.65$ degrees.

b. Let α be an angle formed from line 1 to line 2, then (by corresponding formula)

$$\cos\alpha = \frac{2\cdot 3 + 7\cdot(-5)}{\sqrt{2^2 + 7^2}\cdot\sqrt{3^2 + (-5)^2}} = \frac{6 - 35}{\sqrt{4 + 49}\cdot\sqrt{9 + 25}} = \frac{-29}{\sqrt{53\cdot 34}} = -\frac{29}{\sqrt{1802}}$$

and $\alpha = \arccos\left(\frac{29}{\sqrt{1802}}\right) \approx 47$ degrees.

Answer: found.

Find the equation of the line:
1. having an x-intercept 4 and slope 5
2. through $(5, -8)$ and with intercepts equal
3. through $(-6, 3)$ and with intercepts numerically equal but opposite in sign
4. through $(-5, 3)$ and with x-intercept twice the y-intercept
5. through $(3, 2)$ and having a slope equal to two-thirds of its y-intercept
6. through $(-4, -2)$ and with sum of intercepts 3
7. through $(4, -2)$ and with product of intercepts -18.
8. through $(4, -2)$ and forming with the axes a triangle of area 9 square units.

Solution.

1. We have $y = 5x + b$, and $0 = 5\cdot 4 + b$. It follows that $b = -20$ and the equation of this line: $\underline{5x - y - 20 = 0}$.

2. We have $\frac{x}{a} + \frac{y}{a} = 1$, and $\frac{5}{a} - \frac{8}{a} = 1$. It follows that $a = -3$ and the equation of this line is $-\frac{x}{3} - \frac{y}{3} = 1$ or $\underline{x + y + 3 = 0.}$

3. We have $\frac{x}{a} - \frac{y}{a} = 1$, and $-\frac{6}{a} - \frac{3}{a} = 1$. It follows that $a = -9$ and the equation of this line is $-\frac{x}{9} + \frac{y}{9} = 1$ or $\underline{x - y + 9 = 0.}$

4. We have $\frac{x}{2a} + \frac{y}{a} = 1$, and $-\frac{5}{2a} + \frac{3}{a} = 1$. It follows that $a = \frac{1}{2}$ and the equation of this line is $\underline{x + 2y - 1 = 0.}$

5. We have $y = \frac{2ax}{3} + a$, and $2 = \left(\frac{4}{3} + 1\right)a$. It follows that $a = \frac{6}{7}$ and the equation of this line is $y = \frac{4}{7}x + \frac{6}{7}$ or $\underline{4x - 7y + 6 = 0}$.

6. We have $- + \frac{y}{3-a} = 1$, and $-\frac{4}{} - \frac{2}{3-a} = 1$, $a^2 - a - 12 = 0$. It follows that: *1)* $a = 4$ and the equation of this line is $\frac{}{4} - y = 1$ or $\underline{x - 4y - 4 = 0.}$ or *2)* $a = -3$ and the equation of this line is $-\frac{}{3} + \frac{y}{6} = 1$ or $\underline{2x - y + 6 = 0.}$

7. We have $- - \frac{18y}{} = 1$, and $\frac{4}{} + \frac{36}{} = 1$. It follows that $a = 40$ and the equation of this line is $\frac{}{40} - \frac{18y}{40} = 1$ or $\underline{x - 18y - 40 = 0.}$

8. So as the line is forming with the axes a triangle of area 9 square units, it means that line has the absolute value of product of intercepts be equal to 18. We have the next two cases:

1) $- + \frac{18y}{} = 1$, and $\frac{4}{} - \frac{36}{} = 1$. It follows that $a = -32$ and the equation of this line is $-\frac{}{32} - \frac{18y}{32} = 1$ or $\underline{x + 18y + 32 = 0.}$

2) $- - \frac{18y}{} = 1$, and $\frac{4}{} + \frac{36}{} = 1$. It follows that $a = 40$ and the equation of this line is $\frac{}{40} - \frac{18y}{40} = 1$ or $\underline{x - 18y - 40 = 0.}$

Answer: all was found.