

Answer on Question #57607 – Math – Statistics and Probability

Question

Suppose that certain bolts have length $L=400+X$ mm, where X is a random variable with density

$$f(x)=\frac{3}{4}(1-x^2) \text{ if } -1 \leq x \leq 1 \text{ and } 0 \text{ otherwise.}$$

Determine c so that with a probability of 95% bolt will have the length between $400-c$ and $400+c$.

Solution

$$f(x) = \begin{cases} \frac{3}{4}(1 - x^2), & -1 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\int_{-c}^c f(x) dx = 0.95.$$

$$\int_{-c}^c f(x) dx = \int_{-c}^c \frac{3}{4}(1 - x^2) dx = \frac{3}{4} \left(x - \frac{x^3}{3} \right)_{-c}^c = \frac{3}{4} \left(2c + \frac{(c)^3 - (-c)^3}{3} \right) = \frac{1}{2}(c + c^3) = 0.95$$

The solution of this cubical equation is

$$c = 0.974517.$$

Answer: 0.974517.