

Answer on Question #57570 – Math – Geometry

Question

One side of an equilateral triangle is 18cm. The mid point of its sides are joined to form another triangle whose mid points, In turn are joined to form another triangle. The process is continued indefinitely.

Find:

1. The sum of triangles' sides.
2. Perimeter of all the triangles.
3. Areas of all the triangles.

Solution

If mid points of sides of a triangle are joined, then we form the new triangle that will be similar to the given one.

1. The perimeter of the given equilateral triangle is $18 \times 3 = 54$ cm.
2. The perimeter of each next triangle is 2 times smaller, because its sides are the middle line of the previous triangle. We have a sequence of perimeters 54, 54/2, 54/4, 54/8, 54/16, ... This is an infinite geometric series with the first term $a = 54$ and the common ratio $r = 1/2$. Its sum is

$$S = \frac{a}{1-r} = \frac{54}{1-\frac{1}{2}} = 54 \cdot 2 = 108 \text{ cm}$$

3. The area of the given equilateral triangle is $18^2 \frac{\sqrt{3}}{4} = 81\sqrt{3} \text{ cm}^2$. The area of each next triangle is 4 times smaller. This is an infinite geometric series of areas with the first term $b = 81\sqrt{3}$ and the common ratio $q = 1/4$. Its sum is

$$S = \frac{b}{1-q} = \frac{81\sqrt{3}}{1-\frac{1}{4}} = 27 \cdot 4 \cdot \sqrt{3} = 108\sqrt{3} \text{ cm}^2$$