Question

Assume that the test scores from a college admissions test are normally distributed with a mean of 450 and a standard deviation of 100.

1) What percentage of people taking the test score are between 400 and 500?
2) Suppose someone received a score of 360. What percentage of the people taking the test score better? What percentage score worse?
3) If a particular university will not admit anyone scoring below 480, what percentage of the persons taking the test would be acceptable to the university?

Solution

1) We have that probability of the test score to be less than x is equal to the following:

$$P(x) = \int\limits_{-\infty}^{x} \frac{e^{-\frac{(t-450)^2}{2\cdot100^2}}}{\sqrt{2\pi}\cdot100} dt\ ;$$

So the percentage of people taking the test score between 400 and 500 is equal to the following probability:

$$P(500) - P(400) = \int\limits_{400}^{500} \frac{e^{-\frac{(t-450)^2}{2\cdot100^2}}}{\sqrt{2\pi}\cdot100} dt =$$

$$= \left[z = \frac{t-450}{100}\right] = \int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}\cdot100} \cdot 100 dz = \int\limits_{-\frac{1}{2}}^{\frac{1}{2}} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dz \approx 0.38292;$$

The value was obtained from the table of the Standard Normal Distribution. We conclude that the percentage is approximately equal to 38%.

2) The percentage of the people taking the test score worse than 360 is equal to the following probability:

$$P(360) = \int\limits_{-\infty}^{360} \frac{e^{-\frac{(t-450)^2}{2\cdot100^2}}}{\sqrt{2\pi}\cdot100} dt = \left[z = \frac{t-450}{100}\right] = \int\limits_{-\infty}^{-0.9} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dz \approx 0.18406;$$

So the percentage is approximately equal to 18%. We conclude that the percentage of the people taking the test score better is equal to $100\% - 18\% = 82\%$.

3) We have that the percentage of the persons who would be acceptable is equal to the number of people with score better than 480. It is equal to:

$$1 - P(480) = 1 - \int\limits_{-\infty}^{480} \frac{e^{-\frac{(t-450)^2}{2\cdot100^2}}}{\sqrt{2\pi}\cdot100} dt = \left[z = \frac{t-450}{100}\right] = 1 - \int\limits_{-\infty}^{0.3} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dz \approx 0.38209;$$

So the percentage of the persons taking the test would be acceptable to the university is approximately equal to 38%.