Question

Two altitudes of an isosceles triangle are equal to 20 cm and 30 cm. Determine the possible measures of the base angles of the triangle

Solution
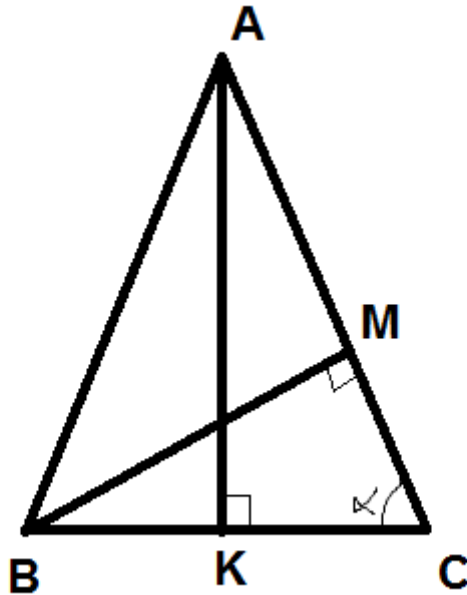
Let we have isosceles triangle ABC, where AB=AC and two altitudes AK and BM.

Here $\tan \alpha = \frac{AK}{KC}$. So $KC = \frac{AK}{\tan \alpha}$.

Also $BC * \sin \alpha = BM$.

$BC = BK + KC = 2 * KC$, because $BK = KC$.

So $2 * KC * \sin \alpha = BM$, $2 * \frac{AK}{\tan \alpha} * \sin \alpha = BM$.

Since $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$, we have $\frac{2*AK*\sin \alpha}{\frac{\sin \alpha}{\cos \alpha}} = BM$, hence $2 * AK * \cos \alpha = BM$.

So $\cos \alpha = \frac{BM}{2*AK}$ and angle $\alpha = \cos^{-1} \frac{BM}{2*AK}$.

If $AK = 20, BM = 30$ then answer will be $\cos^{-1} \frac{3}{4} = 0.72273424$ in radians.

If $AK = 30, BM = 20$ then answer will be $\cos^{-1} \frac{1}{3} = 1.23095942$ in radians.

Answer: base angle is equal to $0.72273424 \, rad$ or $1.23095942 \, rad$.