Answer on Question #57416 – Math – Algebra

Question

Given are: $z_1 = 1 + i$ and $z_2 = \sqrt{3} + i$. Calculate:

a) $z_1 \cdot z_2$,

b) z_1/z_2

and

c) the polar form of both given numbers.

Solution

$z_1 = 1 + i;\ z_2 = \sqrt{3} + i$

a) $z_1 \cdot z_2 = (1 + i) \cdot (\sqrt{3} + i) = 1 \cdot \sqrt{3} + 1 \cdot i + i \cdot \sqrt{3} + i \cdot i = \sqrt{3} + (1 + \sqrt{3})i - 1 =$
$= \sqrt{3} - 1 + (1 + \sqrt{3})i$;

b) $\frac{z_1}{z_2} = \frac{1+i}{\sqrt{3}+i} = \frac{(1+i)\cdot(\sqrt{3}-i)}{(\sqrt{3}+i)\cdot(\sqrt{3}-i)} = \frac{1\cdot\sqrt{3}+1\cdot(-i)+i\sqrt{3}+i\cdot(-i)}{(\sqrt{3})^2-i^2} = \frac{\sqrt{3}-i^2+(\sqrt{3}-1)i}{3+1} =$
$= \frac{\sqrt{3}+1+(\sqrt{3}-1)i}{4} = \frac{\sqrt{3}+1}{4} + \frac{(\sqrt{3}-1)}{4} i$;

c) $z_1 = 1 + i$; $r = \sqrt{1^2 + 1^2} = \sqrt{2}$; $\cos\varphi = 1/\sqrt{2}$, $\sin\varphi = 1/\sqrt{2}$; $\varphi = 45^\circ$;

$$z_1 = r(\cos\varphi + i\ \sin\varphi) = \sqrt{2}\,(\cos 45^\circ + i \sin 45^\circ)$$

$z_2 = \sqrt{3} + i$; $r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$; $\cos\varphi = \sqrt{3}/2$, $\sin\varphi = 1/2$; $\varphi = 30^\circ$;
$z_2 = r(\cos\varphi + i\ \sin\varphi) = 2\,(\cos 30^\circ + i \sin 30^\circ)$.

Remark . If polar form of both numbers is obtained, then multiplication and division in a) and b) can be performed in the following way:

$$\begin{aligned} z_1 \cdot z_2 &= r_1(\cos\varphi_1 + isin\varphi_1) r_2(\cos\varphi_2 + isin\varphi_2) \\ &= r_1 r_2\big(cos(\varphi_1 + \varphi_2) + isin(\varphi_1 + \varphi_2)\big) \\ &= \sqrt{2} \cdot 2\left(cos(45^\circ + 30^\circ) + isin(45^\circ + 30^\circ)\right) = 2\sqrt{2}(cos75^\circ + isin75^\circ) \\ &= 2\sqrt{2}\left(\frac{\sqrt{6} - \sqrt{2}}{4} + i\frac{\sqrt{6} + \sqrt{2}}{4}\right) = \frac{2\sqrt{12} - 2 \cdot 2}{4} + i\frac{2\sqrt{12} + 4}{4} \\ &= (\sqrt{3} - 1) + i(\sqrt{3} + 1) \end{aligned}$$

$$\frac{z_1}{z_2} = \frac{r_1(\cos\varphi_1 + isin\varphi_1)}{r_2(\cos\varphi_2 + isin\varphi_2)} = \frac{r_1}{r_2}\big(cos(\varphi_1 - \varphi_2) + isin(\varphi_1 - \varphi_2)\big) =$$
$$= \frac{\sqrt{2}}{2}\left(cos(45^\circ - 30^\circ) + isin(45^\circ - 30^\circ)\right) = \frac{\sqrt{2}}{2}(cos15^\circ + isin15^\circ) =$$

$$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{6}+\sqrt{2}}{4} + i\,\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \frac{\sqrt{12}+2}{8} + i\,\frac{\sqrt{12}-2}{8} = \frac{\sqrt{3}+1}{4} + i\,\frac{\sqrt{3}-1}{4}.$$

Answer:

a) $\sqrt{3} - 1 + (1 + \sqrt{3})i;$

b) $\frac{\sqrt{3}+1}{4} + \frac{(\sqrt{3}-1)}{4}i$;

c) $1 + \mathrm{i} = \sqrt{2}\,(\cos 45^{\circ} + i \sin 45^{\circ}),$

$\sqrt{3} + i \ = 2\,(\cos 30^{\circ} + i \sin 30^{\circ}).$