Answer on Question #57415 – Math – Calculus

Question

Find the intersection points, if any, for each system of equations. An ellipse with vertices $(0, -5)$ and $(0,5)$ and a minor axis of length 8.

Solution

The equation of an ellipse is:

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

So, the coordinates of the center are $\left(\frac{0+0}{2};\ \frac{-5+5}{2}\right)$=$(0;\ 0)$. The length of a major axis is $2a = 5-(-5) = 10$, hence $a = 5$. It is given that the length of a minor axis is $2b = 8$, hence $b = \frac{8}{2} = 4$.

Therefore, the equation of the ellipse is

$\frac{x^2}{16}+\frac{y^2}{25}=1.$

Setting $x = 0$ in the previous equation gives $\frac{y^2}{25}=1$, hence $y^2 = 25$ and finally $y = 5$ or $y = -5$.

Setting $y = 0$ in the previous equation gives $\frac{x^2}{16}=1$, hence $x^2 = 16$ and finally $x = 4$ or $x = -4$.

The intersection points (x-intercepts and y-intercepts) are $(0; 5)$, $(0; -5)$, $(4; 0)$, $(-4; 0)$.

Answer: $(0; 5)$, $(0; -5)$, $(4; 0)$, $(-4; 0)$.