Answer on Question #57370 – Math – Calculus

Question

Find

$$\int 2x(\boldsymbol{x^2}-7)^3dx.$$

Solution

First method

$$\int 2x(x^2-7)^3dx = |d(x^2-7) = 2xdx| = \int (\boldsymbol{x^2}-7)^3d(\boldsymbol{x^2}-7) = |t = x^2-7| =$$

$= \int t^3dt = \frac{t^4}{4} + c = \frac{1}{4}(x^2-7)^4 + c,$ where c is an arbitrary real constant.

Second method

$$\int 2x(x^2-7)^3dx = \int 2x(x^6 - 3x^4\cdot 7 + 3x^2\cdot 7^2 - 7^3)dx$$
$$= \int 2x(x^6 - 21x^4 + 147x^2 - 343)\,dx$$
$$= \int (2x^7 - 42x^5 + 294x^3 - 686x)dx = 2\frac{x^8}{8} - 42\frac{x^6}{6} + 294\frac{x^4}{4} - 686\frac{x^2}{2} + c$$

$= \frac{x^8}{4} - 7x^6 + 147\frac{x^4}{2} - 343x^2 + c$, where c is an arbitrary real constant.

Answer: $\frac{1}{4}(\boldsymbol{x^2}-7)^4 + c.$