Question

If cos 42 degrees=a then find the value of tan 48 degrees.

Solution

It is known that

$\cot \alpha = \tan(90° − \alpha)$

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So

$\tan 48° = \cot(90° − 48°)$

$\tan 48° = \cot 42°.$

The another identities show that

$$\cot \alpha = \frac{\cos \alpha}{\sin \alpha}$$

and

$\sin^2 \alpha + \cos^2 \alpha = 1,$
hence
$\sin \alpha = \sqrt{1 - \cos^2 \alpha}$, when 0°<α<90°.

From this we can derive:
$$\cot \alpha = \frac{\cos \alpha}{\sqrt{1 - \cos^2 \alpha}} \ .$$
So, we substitute α=42° and get the formula:
$$\cot 42° = \frac{\cos 42°}{\sqrt{1 - \cos^2 42°}}$$
$$\cot 42° = \frac{a}{\sqrt{1 - a^2}} \ .$$
Because

$\tan 48° = \cot 42°,$

the answer will be
$$\tan 48° = \frac{a}{\sqrt{1 - a^2}} \ .$$

Answer: $\tan 48° = \frac{a}{\sqrt{1 - a^2}} \ .$