Answer on Question #57347 - Math – Algebra

Question

How many solutions are there for the system shown below?

$$\begin{cases} x^2 + y^2 = 25 \\ x - y^2 = -5 \end{cases}$$

A: 3
B: 4
C: 2
D: 1

Solution

$$\begin{cases} x^2 + y^2 = 25 \\ x - y^2 = -5 \end{cases} \Rightarrow \begin{cases} y^2 = x + 5 \\ x^2 + y^2 = 25 \end{cases} \Rightarrow \begin{cases} y^2 = x + 5 \\ x^2 + x + 5 = 25 \end{cases} \Rightarrow \begin{cases} y^2 = x + 5 \\ x^2 + x - 20 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} y^2 = x + 5 \\ \left[\begin{array}{l} x = -5 \\ x = 4 \end{array}\right. \end{cases} \Rightarrow \left[\begin{array}{l} \begin{cases} y^2 = x + 5 \\ x = -5 \end{cases} \\ \begin{cases} y^2 = x + 5 \\ x = 4 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} y^2 = 0 \\ x = -5 \end{cases} \\ \begin{cases} y^2 = 9 \\ x = 4 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} y = 0 \\ x = -5 \end{cases} \\ \begin{cases} y = \pm 3 \\ x = 4 \end{cases} \end{array}\right.$$

Answer: the system has 3 solutions (A).

Question

For the system shown below, what are the coordinates of the solution that lies in quadrant 1? Write your answer in the form (a, b) without using spaces.

Solution

$$\begin{cases} x^2 + 4y^2 = 100 \\ 4y - x^2 = -20 \end{cases} \Rightarrow \begin{cases} x^2 = 4y + 20 \\ x^2 + 4y^2 = 100 \end{cases} \Rightarrow \begin{cases} x^2 = 4y + 20 \\ 4y^2 + 4y + 20 = 100 \end{cases} \Rightarrow \begin{cases} x^2 = 4y + 20 \\ y^2 + y - 20 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x^2 = 4y + 20 \\ \left[\begin{array}{l} y = -5 \\ y = 4 \end{array}\right. \end{cases} \Rightarrow \left[\begin{array}{l} \begin{cases} x^2 = 4y + 20 \\ y = -5 \end{cases} \\ \begin{cases} x^2 = 4y + 20 \\ y = 4 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} x^2 = 0 \\ y = -5 \end{cases} \\ \begin{cases} x^2 = 36 \\ y = 4 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} \begin{cases} x = 0 \\ y = -5 \end{cases} \\ \begin{cases} x = \pm 6 \\ y = 4 \end{cases} \end{array}\right.$$

$(x, y) = (6,4)$ is a solution that lies in quadrant 1.

Answer: $(6,4)$.

Question

What are all of the ordered pair solutions for the system of equations shown below?

$$\begin{cases} x^2 + 4y^2 = 100 \\ 4y - x^2 = -20 \end{cases}$$

Solution

From the previous part solutions for the system of equations are

$$(-6,4), (6,4), (0,-5).$$

Answer: $(-6,4), (6,4), (0,-5)$.