Answer on Question #57338 – Math – Statistics and Probability

Question

Suppose a gene in a chromosome is of type A or type B. Assume that the probability that a gene of type A will mutate of type B in one generation is 10^−4 and that a gene of type B will mutate to type A is 10^−6.

(A) What is the transition matrix?

(B) After many generations, what is the probability that the gene will be of type A? or type B? (Find the stationary matrix.)

Solution

(A) $T = \begin{pmatrix} 1 - 10^{-4} & 10^{-4} \\ 10^{-6} & 1 - 10^{-6} \end{pmatrix} = \begin{pmatrix} 0.9999 & 0.0001 \\ 0.000001 & 0.999999 \end{pmatrix}$

(B) $(a, b)T = (a, b) \rightarrow \begin{cases} 0.9999a + 0.000001b = a \\ 0.0001a + 0.999999b = b \end{cases}$ $and\ a + b = 1 \rightarrow$

$\rightarrow \begin{cases} 0.0001a - 0.000001b = 0 \\ a + b = 1 \end{cases} \rightarrow b = 100a \rightarrow 101a = 1 \rightarrow$

$\rightarrow a = \frac{1}{101} \approx 0.00990099,\ b = 1 - a = \frac{100}{101} \approx 0.99009901$.

Stationary matrix $S = \begin{pmatrix} a & a \\ b & b \end{pmatrix} = \begin{pmatrix} \frac{1}{101} & \frac{1}{101} \\ \frac{100}{101} & \frac{100}{101} \end{pmatrix} \approx \begin{pmatrix} 0.00990099 & 0.00990099 \\ 0.99009901 & 0.99009901 \end{pmatrix}$.

Answer: (A) $\begin{pmatrix} 0.9999 & 0.0001 \\ 0.000001 & 0.999999 \end{pmatrix}$; **(B)** $\begin{pmatrix} 0.00990099 & 0.00990099 \\ 0.99009901 & 0.99009901 \end{pmatrix}$.