

Answer on Question #57330 – Math – Trigonometry

Question

In a triangle ABC prove that $\sin(A+B)/2 = \cos C/2$

Solution

It is known that in a triangle, the three interior angles always add to 180° :

$$A+B+C=180^\circ$$

$$A+B=180^\circ - C$$

$$\frac{A+B}{2} = 90^\circ - \frac{C}{2} \quad \Rightarrow \quad 90^\circ - \frac{A+B}{2} = \frac{C}{2} \quad (1)$$

The first identity in the next system holds true due to the well-known identity expressing the sine function in terms of its complement. The second identity in the next system holds true due to identity (1).

$$\begin{cases} \sin \frac{A+B}{2} = \cos \left(90^\circ - \frac{A+B}{2} \right) \\ \cos \left(90^\circ - \frac{A+B}{2} \right) = \cos \left(\frac{C}{2} \right) \end{cases} \Rightarrow \sin \frac{A+B}{2} = \cos \left(\frac{C}{2} \right)$$

Q.E.D.