Answer on Question #57311 - Math – Algebra

Question

1. Solve each system. Show work

7y^2+x^2=64

x+y=4

Solution

$$\begin{cases} 7y^2 + x^2 = 64; \\ x + y = 4. \end{cases}$$

$$\begin{cases} 7y^2 + x^2 = 64; \\ x = 4 - y. \end{cases}$$

Substitute for $x = 4 - y$ into the first equation of the system

$$\begin{cases} 7y^2 + (4 - y)^2 = 64; (1) \\ x = 4 - y. \qquad (2) \end{cases}$$

Solving the first equation of the system

(1) $7y^2 + 16 - 8y + y^2 = 64;$

$8y^2 - 8y - 48 = 0;$

divide by 8:

$y^2 - y - 6 = 0;$

$D = 1 - 4 \cdot (-6) = 25;$

$y_1 = \frac{1-5}{2} = \frac{-4}{2} = -2;$

$y_2 = \frac{1+5}{2} = \frac{6}{2} = 3.$

So

$y_1 = -2; \quad y_2 = 3;$

Plug these values in (2)

(2) $x_1 = 4 - y_1 = 4 - (-2) = 6; \;\; x_2 = 4 - y_2 = 4 - 3 = 1.$

Thus, solutions of the system are $(x_1, y_1) = (6, -2)$ and $(x_2, y_2) = (1, 3)$.

Answer: (6,-2), (1,3).

Question

2. Solve each system. Show work

x^2+y^2+2x+2y=0

x^2+y^2+4x+6y+12=0

Solution

First method

$$\begin{cases} x^2+y^2+2x+2y=0 \\ x^2+y^2+4x+6y+12=0 \end{cases}$$

Subtract the first equation $x^2+y^2+2x+2y=0$ from the second one

$x^2+y^2+4x+6y+12=0$ and obtain

$2x+4y+12=0$

Divide by 2

$x+2y+6=0,$

hence

$x=-2y-6$

Substituting for $x=-2y-6$ into the first equation of the system

$x^2+y^2+2x+2y=0$

solve

$(-2y-6)^2+y^2+2(-2y-6)+2y=0,$

$4y^2+24y+36+y^2-4y-12+2y=0,$

$5y^2+22y+24=0,$

$D=22^2-4\cdot5\cdot24=4,$

$y_1=\frac{-22+2}{2\cdot5}=\frac{-20}{10}=-2,$

$y_2=\frac{-22-2}{2\cdot5}=\frac{-24}{10}=\frac{-12}{5}=-2.4.$

Then

$x_1=-2y_1-6=-2\cdot(-2)-6=4-6=-2,$

$x_2=-2y_2-6=-2\cdot\frac{-12}{5}-6=\frac{24}{5}-6=\frac{24-30}{5}=\frac{-6}{5}=-1.2.$

Thus, solutions of the system are $(x_1,y_1)=(-2,-2)$ and $(x_2,y_2)=\left(-\frac{6}{5},-\frac{12}{5}\right)$.

Second method

$$\begin{cases} x^2+y^2+2x+2y=0; \\ x^2+y^2+4x+6y+12=0; \end{cases}$$

$$\begin{cases} x(x+2)+y(y+2)=0; \\ x(x+4)+y(y+6)+12=0; \end{cases}$$

$$\begin{cases} \frac{1}{2}(x+1)^2+\frac{1}{2}(y+1)^2=1; (1) \\ (x+2)^2+(y+3)^2=1; \qquad (2) \end{cases}$$

(1) and (2) are equations of the circles.

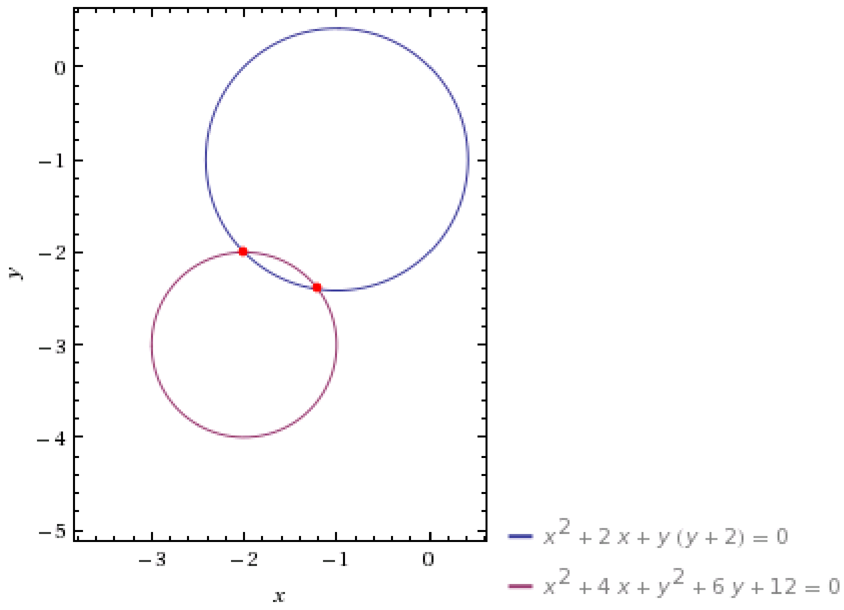

Fig. 1 Graphic solution

$x_1 = -2;\ y_1 = -2;$

$x_2 = -\frac{6}{5};\ y_2 = -\frac{12}{5};$

Answer: (-2, -2), $(-\frac{6}{5}, -\frac{12}{5})$.