Answer on Question #57276 – Math – Analytic Geometry

Question

Which of the following equations represents an ellipse having vertices located at (2,9) and (2,-5) and foci located at (2,5) and (2,-1) ?

A: (x+2)^2 (y+2)^2

---------- + ---------- = 1

40 49

B: (x-2)^2 (y-2)^2

--------- + ---------- = 1

9 49

C: (x-2)^2 (y-2)^2

--------- + ---------- = 1

40 49

D: (x-2)^2 (y-2)^2

---------- + ----------- = 1

49 40

Solution

For ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,\ a > b$

Vertices: $(h + a, k), (h - a, k)$,

Foci: $(h + c, k), (h - c, k),$ where $c^2 = a^2 - b^2$.

For ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,\ a<b$

Vertices: $(h,k+b),(h,k-b)$.

Foci: $(h,k+c),(h,k-c),\text{where}\ \ c^2=b^2-a^2$.

If it is given that vertices are located at (2,9) and (2,-5) and foci are located at (2,5) and (2,-1), then the second type of ellipses ($a<b$) meets these conditions

$$h=2,\qquad \begin{cases} k+b=9 \\ k-b=-5 \end{cases},\ \begin{cases} k+c=5 \\ k-c=-1 \end{cases}\ \rightarrow\ k=2, b=7, c=3,$$

$a^2=b^2-c^2=49-9=40.$

Equation of ellipse is $\frac{(x-2)^2}{40}+\frac{(y-2)^2}{49}=1.$

Answer: C. $\frac{(x-2)^2}{40}+\frac{(y-2)^2}{49}=1.$