Answer on Question #57271– Math – Algebra

List the possible factors for $h(x) = 4x^4 - 5x^3 + 2x^2 - x + 5$. Show work.

Solution

Let $h(x) = 4x^4 - 5x^3 + 2x^2 - x + 5$. A value $x = x_1$ such that $h(x_1) = 0$ is called a root of equation $h(x) = 0$. Finding roots of a polynomial is equivalent to polynomial factorization into factors of degree 1, that is, $h(x) = 4(x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$, where $\alpha, \beta, \gamma, \delta$ are roots of $h(x)$.

By Fundamental Theorem of Algebra, any algebraic equation of degree 4 has exactly 4 roots (real or complex), each root counted according to its multiplicity.

The possible rational roots of a polynomial with integer coefficients are

$$\pm \frac{divisor\ of\ the\ constant\ term}{divisor\ of\ the\ leading\ coefficient}$$

Positive integer divisors of the constant term 5 are 1 and 5.
Positive integer divisors of the leading coefficient 4 are 1, 2, 4.
So the possible rational roots are

$$\pm\mathbf{1}, \pm\mathbf{5}, \pm\frac{\mathbf{1}}{\mathbf{2}}, \pm\frac{\mathbf{5}}{\mathbf{2}}, \pm\frac{\mathbf{1}}{\mathbf{4}}, \pm\frac{\mathbf{5}}{\mathbf{4}}.$$

Substituting these values for x into the expression $4x^4 - 5x^3 + 2x^2 - x + 5$ we shall not obtain zero, hence in fact, these numbers are not roots of $h(x) = 4x^4 - 5x^3 + 2x^2 - x + 5$.

Rational roots cannot be found using Rational Root Test.

Command Solve[4x^4-5x^3+2x^2-x=5==0,x] in Wolfram Mathematica gives approximate complex roots:

-0.44131-0.765181*i*, -0.44131+0.765181*i*, 1.06631-0.681924*i*, 1.06631+0.681924*i*.