

## Answer on Question #57270 – Math – Calculus

### Question

- i) Write a function with the given characteristics, show work: A polynomial with rational coefficients having roots 3, 3 and  $3 - i$
- ii) Write a function with the given characteristics, show work: A rational function with vertical asymptote  $x = 5$  and a horizontal asymptote  $y = \frac{1}{2}$

### Solution

i)

If polynomial with rational coefficients has root  $3 - i$ , it also has a root  $3 + i$  (complex conjugate). The example of polynomial with roots 3, 3,  $3 - i$ ,  $3 + i$  is  $(x - 3)^2(x - (3 - i))(x - (3 + i)) = (x - 3)^2(x - 6x + 10) = x^4 - 12x^3 + 55x^2 - 114x + 90$ .

ii)

The rational function  $f(x)$  has asymptote  $x = 5$  if its denominator is divisible by  $x - 5$ . The rational function  $f(x)$  has horizontal asymptote  $y = \frac{1}{2}$  if  $\lim_{x \rightarrow \infty} \frac{f(x)}{x} = 0$  and  $\lim_{x \rightarrow \infty} f(x) = \frac{1}{2}$ . Hence an example of such a function is  $f(x) = \frac{1}{x-5} + \frac{1}{2} = \frac{x-3}{2(x-5)}$ .

**Answer:** i)  $x^4 - 12x^3 + 55x^2 - 114x + 90$ ; ii)  $\frac{x-3}{2(x-5)}$ .