

Answer on Question #57269 – Math – Algebra

Question

1. Solve the polynomial equation, show work. $x^3 - 7x^2 - x + 7 = 0$

Solution

$$x^3 - 7x^2 - x + 7 = 0.$$

The possible roots of the equation are among the divisors of the constant term 7.

So one of the roots is 7.

Check:

$$7^3 - 7 \cdot 7^2 - 7 + 7 = 0.$$

Find other roots:

$$\frac{x^3 - 7x^2 - x + 7}{x - 7} = x^2 - 1.$$

The roots of $x^2 - 1 = 0$ are 1 and -1, because $(-1)^2 - 1 = 0$ and $1^2 - 1 = 0$.

Thus, the roots of $x^3 - 7x^2 - x + 7 = 0$ are -1, 1, 7.

Answer: the roots of $x^3 - 7x^2 - x + 7 = 0$ are -1, 1, 7.

Question

2. Solve the polynomial equation, show work. $x^4 + x^3 - 6x^2 - 14x - 12 = 0$

Solution

$$x^4 + x^3 - 6x^2 - 14x - 12 = 0.$$

The possible roots of the equation are among the divisors of the constant term -12. One of the roots is -2.

Check:

$$(-2)^4 + (-2)^3 - 6 \cdot (-2)^2 - 14 \cdot (-2) - 12 = 0$$

Find other roots:

$$\frac{x^4 + x^3 - 6x^2 - 14x - 12}{x + 2} = x^3 - x^2 - 4x - 6. \text{ One of the root of } x^3 - x^2 - 4x - 6 = 0 \text{ is 3, because}$$

$$3^3 - 3^2 - 4 \cdot 3 - 6 = 0$$

Find other roots.

$$\frac{x^3 - x^2 - 4x - 6}{x - 3} = x^2 + 2x + 2. \text{ Solve the quadratic equation:}$$

$$D = 2^2 - 4 \cdot 1 \cdot 2 = 4 - 8 = -4, \text{ hence } D < 0, \sqrt{D} = \pm 2i.$$

$$x = \frac{-2 \pm 2i}{2} = -1 \pm i$$

The roots of $x^4 + x^3 - 6x^2 - 14x - 12 = 0$ are -2, 3, -1+i, -1-i.

Answer: the roots of $x^4 + x^3 - 6x^2 - 14x - 12 = 0$ are -2, 3, -1+i, -1-i.