Answer on Question #57267 – Math – Calculus

Question

Determine the equations of the asymptotes, show work: $g(x) = \frac{2x^2+1}{x+3}$.

Solution

1) The graph of $g(x) = \frac{2x^2+1}{x+3}$ has a vertical asymptote $x = -3$, since $\lim_{x\to -3} \frac{2x^2+1}{x+3} = \infty$.

2) If there exist finite limits $k = \lim_{x\to\infty} \frac{g(x)}{x}$ and $b = \lim_{x\to+\infty} (g(x) - kx)$, then the line $y = kx + b$ is an oblique asymptote of the graph for $x \to +\infty$ (in a similar way, one defines an asymptote for $x \to -\infty$).
Check:

$k = \lim_{x\to\infty} \frac{g(x)}{x} = \lim_{x\to\infty} \frac{2x^2+1}{(x+3)x} = 2.$

$b = \lim_{x\to\infty} g(x) - kx = \lim_{x\to\infty} \frac{2x^2+1}{x+3} - 2x == \lim_{x\to\infty} \frac{1-6x}{x+3} = -6.$

Thus, the equation of the oblique asymptote has the form $y = 2x - 6$.

Answer:

$x = -3$,

$y = 2x - 6.$