Answer on Question #57265 – Math – Calculus
Question

1. Sketch the graph of $f(x) = x^4 - 4x^3 - x^2 + 12x - 2$. Identify the extreme values and show work. The graph is scaled 12 tall and 4 wide.

Solution

This function is defined for all $x \in \mathbb{R}$ ($\mathbb{R}$ is its domain), y-intercept is the point $(0; -2)$, because $f(0) = -2$.

Let's calculate the first and the second derivatives to find the extreme values. We have:

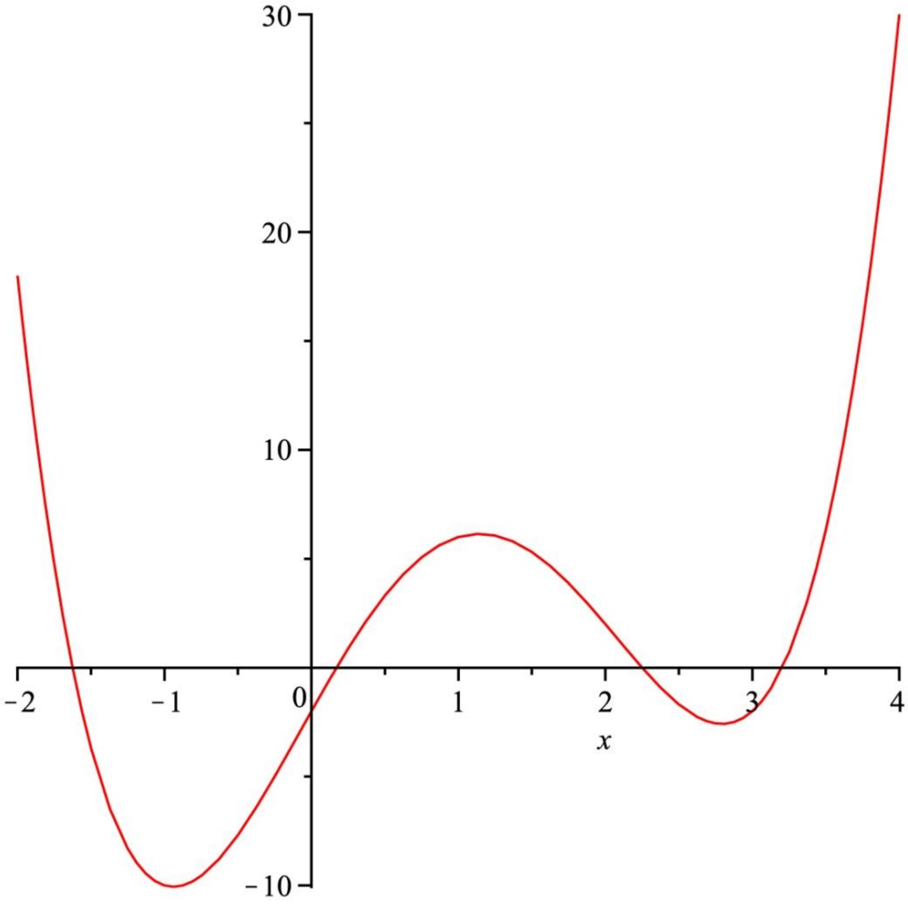
$$f'(x) = 4x^3 - 12x^2 - 2x + 12$$

if $f'(-1) = -2$, $f'(1) = 2$, $f'(2) = -8$, $f'(3) = 6$ then the equation $4x^3 - 12x^2 - 2x + 12 = 0$ has three solutions. Approximately these solutions are $x_1 \approx -0.9$, $x_2 \approx 1.1$, $x_3 \approx 2.8$, then the given function is increasing on $(x_1;\ x_2) \cup (x_3;\ +\infty)$ and it is decreasing on $(-\infty;\ x_1) \cup (x_2;\ x_3)$.

$$f''(x) = 12x^2 - 24x - 2$$

$12x^2 - 24x - 2 = 0$, $D' = 144 + 24 = 168$, $x_5 \approx \frac{1}{12}$, $x_6 \approx 2$ are points of inflection.

We are sketching the graph (for convenience we take a different scale than the given)



Question

2. Sketch the graph of $h(x) = x^2 - 3x - \frac{4}{x} - 4$ Label any important features. State the domain and range show work. The graph is scaled 5 tall and 5 wide.

Solution

The domain of this function is $\mathbb{R}\backslash\{0\}$. The range is $\mathbb{R}$.

The Given function is discontinuous at the point $x = 0$ and $x = 0$ is a vertical asymptote, furthermore

$$\lim_{x \to 0 \mp 0} h(x) = \pm\infty.$$

Let 's calculate the first derivative. We have:

$$f'(x) = 2x - 3 + \frac{4}{x^2}$$

Equation $2x - 3 + \frac{4}{x^2} = 0$ has one solution $x = x^* \approx -0.9$ and $f'(-1) < 0$, $f'(1) > 0$. It means that the given function is increasing on $(x^*;\ 0) \cup (0; +\infty)$ and it is decreasing on $(-\infty;\ x^*)$.

Sketching the graph