

Answer on Question #57233 – Math – Calculus

Question

What is the equation of the hyperbola with vertices at $(0, -4)$ and $(0, 4)$ and foci at $(0, -6)$ and $(0, 6)$

A: $\frac{y^2}{16} - \frac{x^2}{20} = 1$

B: $\frac{x^2}{16} - \frac{y^2}{20} = 1$

C: $\frac{x^2}{16} - \frac{y^2}{36} = 1$

D: $\frac{x^2}{16} - \frac{y^2}{52} = 1$

Solution

Because vertices and foci of hyperbola are located on the y-axis, the center is the point $(0, 0)$, the equation of the hyperbola is

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1,$$

where

$a = 4$, because vertices $(0, a) = (0, 4)$ and $(0, -a) = (0, -4)$ lie on the hyperbola.

Besides,

$c = 6$, because foci are at $(0, 6)$ and $(0, -6)$.

On the other hand,

$$b = \sqrt{c^2 - a^2}.$$

Thus,

$$b = \sqrt{6^2 - 4^2} = \sqrt{36 - 16} = \sqrt{20} = 2\sqrt{5}$$

and finally

$$\frac{y^2}{4^2} - \frac{x^2}{(\sqrt{20})^2} = 1,$$

$$\frac{y^2}{16} - \frac{x^2}{20} = 1$$

Answer: A: $\frac{y^2}{16} - \frac{x^2}{20} = 1$.