

### Answer on Question #57221 – Math – Geometry

1. The radius of a regular decagon is 6m. What is the length of its apothem?

#### Solution

Definition: Apothem is a line segment from the center of a regular polygon to the midpoint of a side.

If you know the radius (distance from the center to a vertex), then the length of apothem is given by

$$apothem = r \cos\left(\frac{180}{n}\right),$$

where

$r$  is the radius of the polygon,

$n$  is the number of sides,

$\cos$  is the cosine function calculated in **degrees**.

$$apothem = 6 \cos \frac{180}{10} = 6 \cos 18 = \mathbf{5.71m}.$$

**Answer:** 5.71 m.

2. The area of a triangle with sides of length 10 in. and 17 in. and an included angle of  $113^\circ$  is equal to the area of a regular heptagon. Determine the length of each side of the heptagon.

#### Solution

$$A_{triangle} = \frac{1}{2} 10 \cdot 17 \sin 113^\circ = 75.74$$

By definition, all sides of a regular polygon are equal in length.

If you know the length of one of the sides, then the area is given by the formula:

$$area = \frac{s^2 N}{4 \tan\left(\frac{180}{N}\right)} \quad (1)$$

where  $s$  is the length of any side,

$N$  is the number of sides,

$\tan$  is the tangent function calculated in degrees.

Given the area of a regular heptagon is

$$A_{heptagon} = A_{triangle} = 75.74.$$

On the other hand, by the formula (1)

$$A_{heptagon} = \frac{7s^2}{4 \tan\left(\frac{180}{7}\right)} \Rightarrow s = \sqrt{\frac{4}{7} A_{heptagon} \tan\left(\frac{180}{7}\right)} = \sqrt{\frac{4}{7} \cdot 75.74 \cdot 0.48} = \mathbf{20.77 in}.$$

$s = \mathbf{20.77 in}$ , where  $s$  is the length of each side of the heptagon.

**Answer:** 20.77 in.

3. Determine the area of the waste material in cutting out the largest circle (diameter is 23 cm) from a regular decagon.

**Solution**

The area of the waste material is

$$A_{\text{waste material}} = A_{\text{decagon}} - A_{\text{circle}}$$

The area of the circle is

$$A_{\text{circle}} = \left(\frac{23}{2}\right)^2 \pi = 415.27$$

If you know the apothem, or inradius, then the area of a regular polygon is given by

$$\text{area} = A^2 N \tan\left(\frac{180}{N}\right)$$

where

$A$  is the length of the apothem (inradius),

$N$  is the number of sides,

$\tan$  is the tangent function calculated in degrees.

The area of the regular decagon is

$$A_{\text{decagon}} = \left(\frac{23}{2}\right)^2 \cdot 10 \cdot \tan 18 = 429.71$$

Then

$$A_{\text{waste material}} = 429.71 - 415.27 = \mathbf{14.44 \text{ cm}^2}.$$

**Answer:** 14.44  $\text{cm}^2$ .