Answer on Question #57151 – Math – Calculus

If $f(x,y)=\tan^{-1}\frac{2xy}{x^2+y^2}$, then prove that $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$.

Solution

This question is confusing.

If $f(x,y)=\tan^{-1}\frac{2xy}{x^2+y^2}=\arctan\frac{2xy}{x^2+y^2}$ then

$$\frac{\partial f}{\partial x}=\frac{1}{1+\left(\frac{2xy}{x^2+y^2}\right)^2}\cdot\frac{2y(x^2+y^2)-4x^2y}{(x^2+y^2)^2}=\frac{2y(y^2-x^2)}{(x^2+y^2)^2+4x^2y^2}$$

$$\frac{\partial^2 f}{\partial x^2}=\frac{-4xy(x^4+6x^2y^2+y^4)-2y(y^2-x^2)(4x^3+12xy^2)}{(x^4+6x^2y^2+y^4)^2}=\frac{-4xy(x^4-2x^2y^2-7y^4)}{(x^4+6x^2y^2+y^4)^2}$$

and respectively

$$\frac{\partial^2 f}{\partial y^2}=\frac{-4xy(y^4-2x^2y^2-7x^4)}{(x^4+6x^2y^2+y^4)^2}$$

Therefore,

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=\frac{4xy(x^4-2x^2y^2-7y^4)}{(x^4+6x^2y^2+y^4)^2}+\frac{4xy(x^4-2x^2y^2-7y^4)}{(x^4+6x^2y^2+y^4)^2}=\frac{-8xy(3x^4+2x^2y^2+3y^4)}{(x^4+6x^2y^2+y^4)^2}\neq 0.$$

Fixed statement of question

If $f(x,y)=\tan^{-1}\frac{2xy}{x^2-y^2}$, then prove that $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$.

Solution. If $f(x,y)=\tan^{-1}\frac{2xy}{x^2-y^2}=\arctan\frac{2xy}{x^2-y^2}$ then

$$\frac{\partial f}{\partial x}=\frac{1}{1+\left(\frac{2xy}{x^2-y^2}\right)^2}\cdot\frac{2y(x^2-y^2)-4x^2y}{(x^2-y^2)^2}=\frac{-2y(y^2+x^2)}{(x^2+y^2)^2}=\frac{-2y}{(x^2+y^2)}$$

$$\frac{\partial^2 f}{\partial x^2}=\frac{4xy}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}=\frac{1}{1+\left(\frac{2xy}{x^2-y^2}\right)^2}\cdot\frac{2x(x^2-y^2)+4y^2x}{(x^2-y^2)^2}=\frac{2x(y^2+x^2)}{(x^2+y^2)^2}=\frac{2x}{(x^2+y^2)}$$

$$\frac{\partial^2 f}{\partial y^2}=-\frac{4xy}{(x^2+y^2)^2}$$

and respectively

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0.$$
