Answer on Question #57135 – Math – Complex Analysis

Find locus of points in plane satisfying given conditions.

(i) $|z-1|=6$

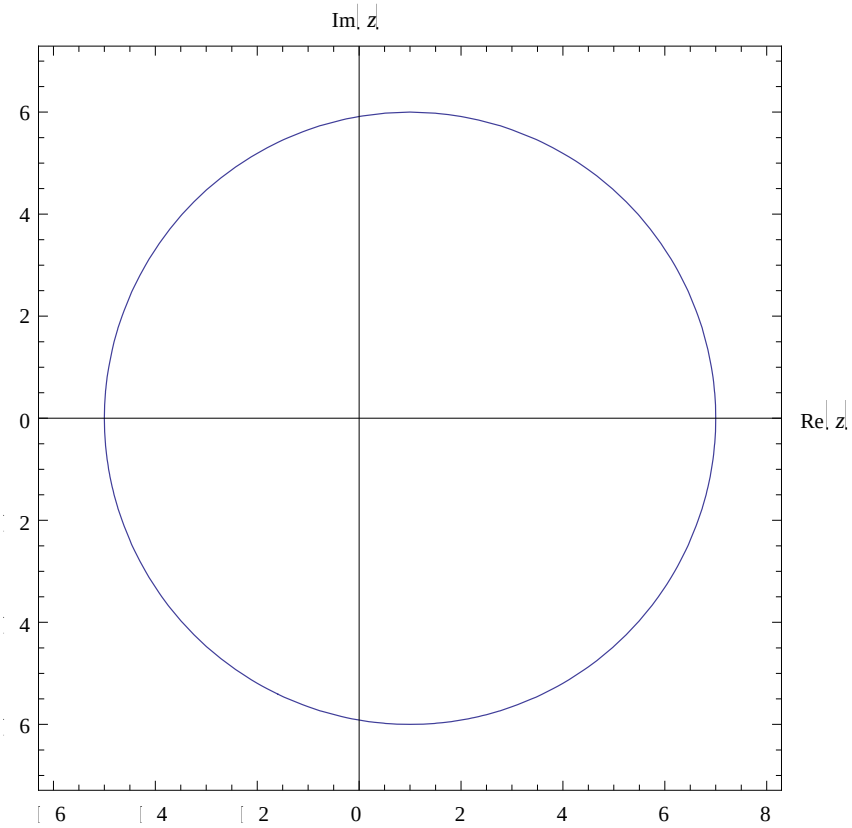
(ii) $|z+3|+|z+1|=4$

(iii) $\operatorname{Arg} z=\pi/3$

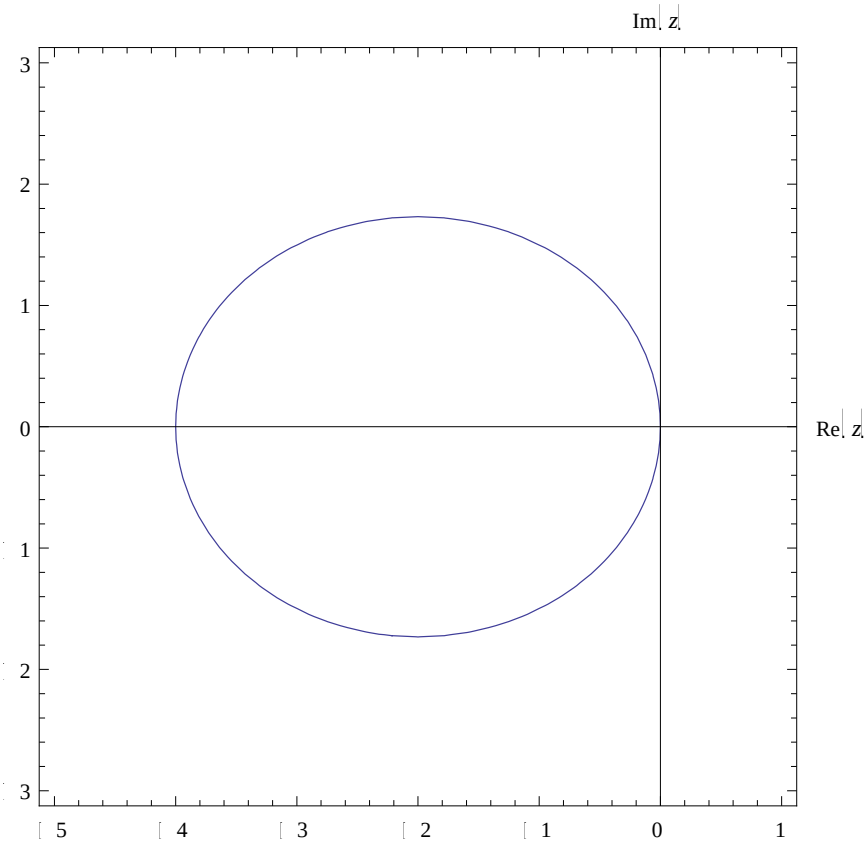
(iv) $\operatorname{Arg}(z-1)=-3\pi/4$

Solution

(i) $|z-1|=6$ represents a circle of radius 6 centered at point $(1,0)$.



(ii) $|z+3|+|z+1|=4$ represents an ellipse with foci at $(-3,0)$ and $(-1,0)$

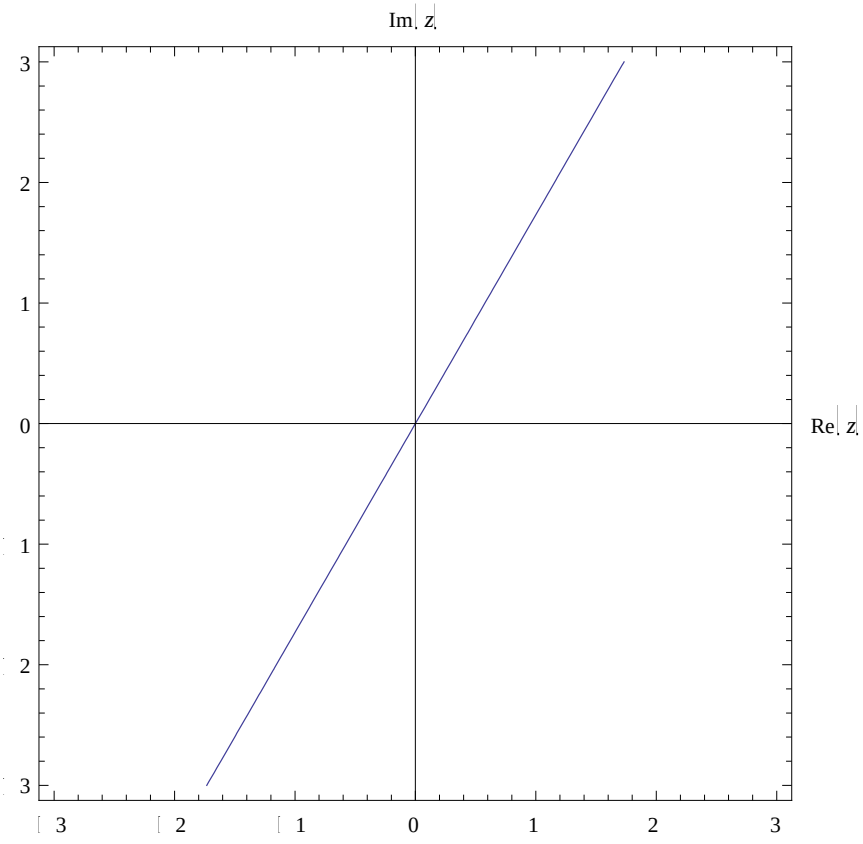


(iii) $\text{Arg}\, z = \pi/3$ represents a straight line of the form

$$y = kx,$$

where $k = \tan \pi/3 = \sqrt{3},\ x = \text{Re}(z),\ y = \text{Im}(z).$

$$y = \sqrt{3}x$$



(iv) $\mathrm{Arg}(z-1) = -3\,\pi/4$ represents a line of the form

$$y = k(x-1),$$

where $k = \tan(-\frac{3\pi}{4}) = -1/\sqrt{2}$, $x = \mathrm{Re}(z)$, $y = \mathrm{Im}(z)$.

$$y = -\frac{x-1}{\sqrt{2}}$$