Answer on Question #57118 – Math – Calculus

If $f(x,y) = \frac{x^2+y^2}{xy}$

Then prove that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

Solution

$$\frac{\partial f}{\partial x} = \left(\frac{x^2+y^2}{xy}\right)'_x = \left(\frac{x}{y} + \frac{y}{x}\right)'_x = \frac{1}{y} - \frac{y}{x^2}$$

$$\frac{\partial f}{\partial y} = \left(\frac{x^2+y^2}{xy}\right)'_y = \left(\frac{x}{y} + \frac{y}{x}\right)'_y = -\frac{x}{y^2} + \frac{1}{x}$$

$$\frac{\partial^2 f}{\partial x^2} = \left(\frac{x^2+y^2}{xy}\right)''_{xx} = \left(\frac{\partial f}{\partial x}\right)'_x = \left(\frac{1}{y} - \frac{y}{x^2}\right)'_x = 0 + \frac{2y}{x^3} = \frac{2y}{x^3}$$

$$\frac{\partial^2 f}{\partial y^2} = \left(\frac{x^2+y^2}{xy}\right)''_{yy} = \left(\frac{\partial f}{\partial y}\right)'_y = \left(\frac{1}{x} - \frac{x}{y^2}\right)'_y = 0 + \frac{2x}{y^3} = \frac{2x}{y^3}$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{2y}{x^3} + \frac{2x}{y^3} = \frac{2y^4 + 2x^4}{x^3 y^3}$$

$2y^4 + 2x^4 = 0$ if and only if $(x,y) = (0,0)$, but expression $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ is not defined when $(x,y) = (0,0)$, because we cannot divide by zero.

Thus,

$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \neq 0.$

Answer:

$\frac{\partial^2 f}{\partial x^2} = \frac{2y}{x^3}$ and $\frac{\partial^2 f}{\partial y^2} = \frac{2x}{y^3}$

$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \neq 0.$