Answer on Question #57115 – Math – Calculus

Question

8. Which function has a removable discontinuity?

A: $f(x) = 5x/1 - x^2$
B: $g(x) = 2x - 1/x$
C: $h(x) = x^2 - x - 2/x + 1$
D: $p(x) = x/x^2 - x - 2$

Solution

Function $h(x) = \frac{x^2 - x - 2}{x + 1} = x - 2$ has a removable discontinuity at $x = -1$.

Answer: C: $h(x) = (x^2 - x - 2)/(x + 1)$,

Question

9. What is the equation of the oblique asymptote?

$$h(x) = \frac{x^2 - 3x - 4}{x + 2}$$

A: $y = x + 4$
B: $y = x - 5$
C: $y = x$
D: $y = x - 4$

Solution

$y = kx + b$ is the equation of the oblique asymptote,

where

$k = \lim_{x \to \pm\infty} \frac{f(x)}{x}$,

$b = \lim_{x \to \pm\infty} (f(x) - kx)$.

$$k = \lim_{x\to\pm\infty} \frac{x^2 - 3x - 4}{(x+2)x} =$$

$$= \lim_{x\to\pm\infty} \frac{x^2 - 3x - 4}{x^2 + 2x}$$

$$= \lim_{x\to\pm\infty} \frac{x^2/x^2 - 3x/x^2 - 4/x^2}{x^2/x^2 + 2x/x^2} = \lim_{x\to\pm\infty} \frac{1 - 3/x - 4/x^2}{1 + 2/x}$$

$$= \frac{\lim_{x\to\pm\infty} (1 - 3/x - 4/x^2)}{\lim_{x\to\pm\infty} (1 + 2/x)} = \frac{\lim_{x\to\pm\infty} (1) - \lim_{x\to\pm\infty} \left(\frac{3}{x}\right) - \lim_{x\to\pm\infty} \left(\frac{4}{x^2}\right)}{\lim_{x\to\pm\infty} (1) + \lim_{x\to\pm\infty} (2/x)} = \frac{1 - 0 - 0}{1 + 0} =$$

$$= \frac{1}{1} = 1$$

$$b = \lim_{x\to\pm\infty} \left(\frac{x^2 - 3x - 4}{x + 2} - x\right) = \lim_{x\to\pm\infty} \left(\frac{x^2 - 3x - 4 - x(x+2)}{x + 2}\right)$$

$$= \lim_{x\to\pm\infty} \left(\frac{x^2 - 3x - 4 - x^2 - 2x}{x + 2}\right) = \lim_{x\to\pm\infty} \left(\frac{-5x - 4}{x + 2}\right) = \lim_{x\to\pm\infty} \left(\frac{-5x/x - 4/x}{x/x + 2/x}\right)$$

$$= \lim_{x\to\pm\infty} \left(\frac{-5 - 4/x}{1 + 2/x}\right) = \frac{\lim_{x\to\pm\infty} (-5 - 4/x)}{\lim_{x\to\pm\infty} (1 + 2/x)} = \frac{\lim_{x\to\pm\infty} (-5) - \lim_{x\to\pm\infty} (4/x)}{\lim_{x\to\pm\infty} (1) + \lim_{x\to\pm\infty} (2/x)} = \frac{-5 - 0}{1 + 0}$$

$$= \frac{-5}{1} = -5$$

$y = x - 5$ is the equation of the oblique asymptote in this question.

Answer: B: $y = x - 5$

Question

10. Which function has the following characteristics?
A vertical asymptote as $x = 3$
A horizontal asymptote at $y = -\frac{1}{2}$

A: $y = \frac{-x}{2x^2 - x - 12}$
B: $y = \frac{-x^2}{2x^2 - 2x - 12}$
C: $y = \frac{x}{2x^2 - x - 3}$
D: $y = \frac{-1}{2x^2 - 2x - 6}$

Solution

To find vertical asymptotes of $y = \frac{-x^2}{2x^2 - 2x - 12}$, solve the equation

$$2x^2 - 2x - 12 = 0$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2)=0$$

$$x=3 \text{ or } x=-2$$

Thus, $x=3$ and $x=-2$ are vertical asymptotes for $y = \frac{-x^2}{2x^2-2x-12}$.

To find vertical asymptotes of $y = \frac{-x^2}{2x^2-2x-12}$, calculate

$$\lim_{x\to\infty}\frac{-x^2}{2x^2-2x-12} = \lim_{x\to\infty}\frac{-x^2/x^2}{\frac{2x^2}{x^2}-\frac{2x}{x^2}-12/x^2} = \lim_{x\to\infty}\frac{-1}{2-\frac{2}{x}-\frac{12}{x^2}} = \frac{\lim_{x\to\infty}(-1)}{\lim_{x\to\infty}\left(2-\frac{2}{x}-\frac{12}{x^2}\right)} = \frac{-1}{\lim_{x\to\infty} 2-\lim_{x\to\infty}\frac{2}{x}-\lim_{x\to\infty}\frac{12}{x^2}} = \frac{-1}{2-0-0} =$$
$$= -\frac{1}{2}.$$

Thus, $y = -\frac{1}{2}$ is vertical asymptote for $y = \frac{-x^2}{2x^2-2x-12}$.

Answer: B: $y = \frac{-x^2}{2x^2-2x-12}$.