Answer on Question#57101 –Math– Calculus

Prove that $\iint (x+y)dxdy = \frac{241}{60}$ at $x = 1$ to $(4-y)^{\frac{1}{2}}$ and $y = 0$ to 3.

Solution.

Let us find the value of the double integral over the specified bounds

$$\iint (x+y)dxdy = \int_0^3 dy \int_1^{\sqrt{4-y}} (x+y)dx =$$

$$= \int_0^3 \left(\frac{x^2}{2} + xy\right)\Bigg|_1^{\sqrt{4-y}} dy = \int_0^3 \left(\frac{4-y}{2} + y\sqrt{4-y} - \frac{1}{2} - y\right) dy = \int_0^3 \left(\frac{3}{2} - \frac{3}{2}y + y\sqrt{4-y}\right) dy =$$

$$= \left(\frac{3y}{2} - \frac{3y^2}{4}\right)\Bigg|_0^3 + \int_0^3 y\sqrt{4-y}dy = \frac{9}{2} - \frac{27}{4} + \int_0^3 y\sqrt{4-y}dy = -\frac{9}{4} + \int_0^3 y\sqrt{4-y}dy$$

Find the integral $\int_0^3 y\sqrt{4-y}dy$ using the integration by part

$$\int_0^3 y\sqrt{4-y}dy = \begin{vmatrix} u = y & dv = \sqrt{4-y} \\ du = dy & v = -\frac{2\sqrt{(4-y)^3}}{3} \end{vmatrix} = -\frac{2y\sqrt{(4-y)^3}}{3}\Bigg|_0^3 + \frac{2}{3}\int_0^3 \sqrt{(4-y)^3}dy =$$

$$= -2 + 0 + \frac{2}{3}\left(-\frac{2}{5}\right)\sqrt{(4-y)^5}\Bigg|_0^3 = -2 - \frac{4}{15}\cdot 1 + \frac{4}{15}\cdot 32 = -\frac{34}{15} + \frac{128}{15} = \frac{94}{15}$$

As result

$$\iint (x+y)dxdy = -\frac{9}{4} + \frac{94}{15} = \frac{-135+376}{60} = \frac{241}{60}$$

Proved that

$$\iint (x+y)dxdy = \int_0^3 dy \int_1^{\sqrt{4-y}} (x+y)dx = \frac{241}{60}$$