Answer on Question #57098 - Math – Calculus

Q. Prove that ∬▒〖(x+y)dxdy=241/60〗 at x=1 to √(4-y) and y=0 to 3.

Solution

$$\iint (\mathrm{x}+\mathrm{y})\mathrm{dxdy} = \int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dx\,dy = \int_0^3 \left(\frac{x^2}{2} + xy\right)\Big|_1^{\sqrt{4-y}} dy =$$

$$= \int_0^3 \left(\frac{4-y}{2} + \sqrt{4-y}y - \frac{1}{2} - y\right) dy = \int_0^3 \left(-\frac{3}{2}y + \frac{3}{2} + y\sqrt{4-y}\right) dy$$

$$\int y\sqrt{4-y}dy = \begin{vmatrix} 4-y=t \\ y=4-t \\ dy=-dt \end{vmatrix} = \int \sqrt{t}(t-4)dt = \int \left(t^{\frac{3}{2}} - 4t^{\frac{1}{2}}\right) dt =$$

$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 4\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{5}t^{\frac{5}{2}} - \frac{8}{3}t^{\frac{3}{2}} = \frac{2}{5}(4-y)^{\frac{5}{2}} - \frac{8}{3}(4-y)^{\frac{3}{2}}$$

So

$$\int_0^3 \left(-\frac{3}{2}y + \frac{3}{2} + y\sqrt{4-y}\right) dy = \left(-\frac{3y^2}{4} + \frac{3y}{2} + \frac{2}{5}(4-y)^{\frac{5}{2}} - \frac{8}{3}(4-y)^{\frac{3}{2}}\right)\Big|_0^3 = \frac{241}{60}$$

Answer: $\frac{241}{60}$.