

Answer on Question #57094 – Math – Calculus

Prove:

$$\iint (x + y) dx dy = \frac{241}{60}$$

**Solution**

$$\begin{aligned} \iint (x + y) dx dy &= \int_0^3 dy \int_1^{\sqrt{4-y}} (x + y) dx = \int_0^3 dy \left( \frac{x^2}{2} + x * y \right) \Big|_1^{\sqrt{4-y}} \\ &= \int_0^3 \left( \frac{4-y}{2} + y\sqrt{4-y} - \frac{1}{2} - y \right) dy \\ &= \int_0^3 \left( \frac{3}{2} - \frac{3y}{2} + y\sqrt{4-y} - 4\sqrt{4-y} + 4\sqrt{4-y} \right) dy \\ &= \int_0^3 \left( \frac{3}{2} - \frac{3y}{2} - (4-y)\sqrt{4-y} + 4\sqrt{4-y} \right) dy = \\ &= \int_0^3 \left( \frac{3}{2} - \frac{3y}{2} - \sqrt{(4-y)^3} + 4\sqrt{4-y} \right) dy \\ &= \left( \frac{3}{2}y - \frac{3y^2}{4} + \frac{2}{5}\sqrt{(4-y)^5} - \frac{8}{3}\sqrt{(4-y)^3} \right) \Big|_0^3 = \frac{9}{2} + \frac{27}{4} + \frac{2}{5} - \frac{8}{3} - \frac{64}{5} + \frac{64}{3} \\ &= \frac{241}{60} \end{aligned}$$