Answer on Question #57093 – Math – Calculus

Use the convolution result in the frequency domain to obtain $\mathcal{F}\{H(t)\sin\omega_0 t\}$.

Solution

We use the standard definition of Fourier transform as

$$\mathcal{F}\{f(t)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{i\omega t} dt.$$

It is accepted in many textbooks a well as in Wolfram Mathematica. The Fourier transforms of Heaviside step function and sine-function are supposed to be known:

$$F_1(\omega) = \mathcal{F}\{\sin\omega_0 t\} = i\sqrt{\frac{\pi}{2}}\left(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right)$$

$$F_2(\omega) = \mathcal{F}\{H(t)\} = \frac{i}{\sqrt{2\pi}\omega} + \sqrt{\frac{\pi}{2}}\delta(\omega).$$

Then by the convolution property in the frequency domain we obtain

$$\mathcal{F}\{H(t)\sin\omega_0 t\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F_1(\omega')\, F_2(\omega-\omega') d\omega' =$$

$$= -\frac{1}{2\sqrt{2\pi(\omega-\omega_0)}} + \frac{1}{2\sqrt{2\pi(\omega+\omega_0)}} + \frac{i}{2}\sqrt{\frac{\pi}{2}}\delta(\omega-\omega_0) - \frac{i}{2}\sqrt{\frac{\pi}{2}}\delta(\omega+\omega_0).$$