Answer on Question #57078 – Math – Calculus

Question

Prove that
∬_01^(1√(4-1))▒〖(x+y)dxdy=241/60〗

Solution

Notice that the statement of question is wrong.

$$\int_a^b\int_c^d (x+y)dxdy = \int_a^b\int_c^d xdxdy + \int_a^b\int_c^d ydxdy = \left(\frac{x^2}{2}\right)_c^d y\Big|_a^b + \left(\frac{y^2}{2}\right)_a^b x\Big|_c^d = \frac{d^2-c^2}{2}(b-a) + \frac{b^2-a^2}{2}(d-c) =$$

$$= \frac{(b-a)(d-c)}{2}(d+c+b+a) = \frac{(b-a)(d-c)}{2}(a+b+c+d),$$

$$\iint (x+y)dxdy = \int_0^3\int_1^{\sqrt{4-y}} (x+y)\,dx\,dy = \int_0^3 \left(\frac{x^2}{2}+xy\right)\Big|_1^{\sqrt{4-y}} dy =$$

$$= \int_0^3 \left(\frac{4-y}{2} + \sqrt{4-y}y - \frac{1}{2} - y\right) dy = \int_0^3 \left(-\frac{3}{2}y + \frac{3}{2} + y\sqrt{4-y}\right) dy$$

Evaluate

$$\int y\sqrt{4-y}dy = \begin{vmatrix} 4-y=t \\ y=4-t \\ dy=-dt \end{vmatrix} = \int \sqrt{t}(t-4)dt = \int \left(t^{\frac{3}{2}} - 4t^{\frac{1}{2}}\right)dt =$$

$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 4\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{5}t^{\frac{5}{2}} - \frac{8}{3}t^{\frac{3}{2}} = \frac{2}{5}(4-y)^{\frac{5}{2}} - \frac{8}{3}(4-y)^{\frac{3}{2}}$$

So

$$\int_0^3 \left(-\frac{3}{2}y + \frac{3}{2} + y\sqrt{4-y}\right) dy = \left(-\frac{3y^2}{4} + \frac{3y}{2} + \frac{2}{5}(4-y)^{\frac{5}{2}} - \frac{8}{3}(4-y)^{\frac{3}{2}}\right)\Big|_0^3 = \frac{241}{60}$$