Answer on Question #57073 – Math – Calculus

Question

Use the time-shift property to calculate the Fourier transform of the double pulse defined by $f(t) = \begin{cases} 1, 1 \le |t| \le 2, \\ 0, otherwise. \end{cases}$

Solution

Consider the next functions:

$$f_1(t) = \begin{cases} 1, -2 \le t \le -1, \\ 0, \ otherwise, \end{cases}$$

$$f_2(t) = \begin{cases} 1, \quad 1 \le t \le 2, \\ 0, otherwise, \end{cases}$$

and

$$g(t) = \begin{cases} 1, -0.5 \le t \le 0.5, \\ 0, \ otherwise. \end{cases}$$

Obviously, $f_1(t) = g(t + 1.5)$; $f_2(t) = g(t - 1.5)$.

The Fourier transform of g is

$$\hat{g}(y) = \int_{-0.5}^{0.5} e^{-2\pi ity} dt = \{g \ is \ even\} = 2\int_0^{0.5} \cos(2\pi ty) dt = \frac{\sin \pi y}{\pi y}.$$

Since $f(t) = f_1(t) + f_2(t)$ we have:

$$\hat{f}(y) = \widehat{f_1}(y) + \widehat{f_2}(y).$$

The time-shift property implies:

$$\hat{f}(y) = e^{-2\pi i\cdot(-1.5)y}\hat{g}(y) + e^{-2\pi i\cdot 1.5y}\hat{g}(y) = \frac{\sin \pi y}{\pi y}\left(e^{3\pi iy} + e^{-3\pi iy}\right) = 2\frac{\sin(\pi y)\cdot\cos(3\pi y)}{\pi y}.$$

Answer: $\hat{f}(y) = 2\frac{\sin(\pi y)\cdot\cos(3\pi y)}{\pi y}$.