Answer on Question #57060 – Math – Calculus

if x+y+z=81 then find the least value of 1/x3 +1/y3 +1/z3

Solution

We use the method of Lagrange multipliers.

Form a function $F(x,y,z,\lambda)=1/x^3+1/y^3+1/z^3+\lambda(x+y+z-81)$.

Equating the partial derivatives to zero we construct a system to determine stationary points:

$$\begin{cases} \dfrac{\partial F}{\partial x}=-\dfrac{3}{x^4}+\lambda=0 \\ \dfrac{\partial F}{\partial y}=-\dfrac{3}{y^4}+\lambda=0 \\ \dfrac{\partial F}{\partial z}=-\dfrac{3}{z^4}+\lambda=0 \\ \dfrac{\partial F}{\partial \lambda}=x+y+z-81 \end{cases},$$

where ,evidently, $\lambda>0$, because in case of $\lambda=0$ the system has no solution.

From the first three equations it follows that

$$x=y=z=\sqrt[4]{\frac{3}{\lambda}} \quad . \qquad (1)$$

Substituting it into the last one we get $3\cdot\sqrt[4]{\frac{3}{\lambda}}=81$ *or* $\sqrt[4]{\frac{3}{\lambda}}=27$. Using the previous equality and (1) we find $x=y=z=27$. So, the function has a single critical point and $F(27,27,27,\lambda)=\frac{3}{27^3}=\frac{1}{6561}$.

Since ,for example, $F(1,40,40,\lambda)=1+\frac{1}{40^3}+\frac{1}{40^3}>1>\frac{1}{6561}$, the point (27,27,27,λ) is the point of minimum. That is $\min\limits_{x+y+z-81=0}\left\{1/x^3+1/y^3+1/z^3\right\}=\frac{1}{6561}$.

Answer: the least value of $\frac{1}{x^3}+\frac{1}{y^3}+\frac{1}{z^3}$ is $\frac{1}{6561}$.