Answer on Question #57032 – Math – Algebra

Question

6. Which of the following functions have a domain of (-∞,∞) and a range of (2,∞).

Check all that apply.

f(x) = 0.25^x -2
f(x) = 3^x -2
f(x) = 0.25^x + 2
f(x) = 3^x + 2

Solution

f(x) = 0.25^x -2 $\Rightarrow$ $f(-\infty) = +\infty,\ f(x) \to -2$ as $x \to +\infty, f(x) > -2$

f(x) = 3^x -2 $\Rightarrow$ $f(-\infty) = +\infty,\ f(+\infty) = +\infty, f(x) \geq 2$

f(x) = 0.25^x + 2 $\Rightarrow$ $f(-\infty) = +\infty,\ f(x) \to 2$ as $x \to +\infty, f(x) > 2$

f(x) = 3^x + 2 $\Rightarrow$ $f(-\infty) = +\infty,\ f(+\infty) = +\infty, f(0) = 2, f(x) \geq 2$

Answer: f(x) = 0.25^x + 2.

Question

7. What is the sum of the first five terms of the geometric sequence in which a1 = 3 and r = 1/3?

Express your answer as an improper fraction using the slash(/) key and no spaces._____

Solution

$$S_n = a_1 * \frac{(1 - r^n)}{1 - r}$$

$$S_5 = a_1 * \frac{(1 - r^5)}{1 - r} = 3 * \frac{\left(1 - \frac{1}{3^5}\right)}{1 - \frac{1}{3}} = 3 * \frac{\left(1 - \frac{1}{243}\right)}{\frac{2}{3}} = \frac{\left(1 - \frac{1}{243}\right)}{2} = \frac{\frac{242}{243}}{2} = \frac{121}{243}$$

Answer: 121/243.

Question

8. The original purchase price of a car is $12,000. Each year, its value depreciates by 5%. Three years after the purchase, what is the value of the car?

A: $1.50

B: $13,891.50

C: $10,830

D: $10,288.50

Solution

$$P_n = P_0 * (1 - r)^n$$

$$P_3 = 12000 * (1 - 0.05)^3 = 10288.50$$

Answer: D: $10,288.50.

Question

9. Which of the following sequences are geometric?

Check all that apply

6,18,54,162,486

2,5,8,11,14,17

-4,-2,-1,-0.5,-0.25,-0.125

2,3,5,8,13,21

Solution

$a_1 = 6$, $a_2 =$18, $a_3 =$54, $a_4 =$162, $a_5 =$486 is a geometric progression with $a_1 = 6, r = 3$, because

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \frac{a_5}{a_4} = 3$$

$a_1 =$ -4, $a_2 =$-2, $a_3 =$-1, $a_4 =$-0.5, $a_5 =$-0.25 is a geometric progression with $a_1 = -4$,

$r = \frac{1}{2}$, because

$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \frac{a_5}{a_4} = \frac{1}{2}$

In sequences 2,5,8,11,14,17 and 2,3,5,8,13,21 equalities $r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \frac{a_5}{a_4}$ do not hold true, therefore these sequences are not geometric.

Answer: 6,18,54,162,486 and -4,-2,-1,-0.5,-0.25,-0.125