Answer on Question #57026 – Math – Calculus

Question

Find $L^{-1}\left\{\frac{z}{z^2+2z-3}\right\}$.

Solution

$$\frac{z}{z^2+2z-3}=\frac{z}{(z-1)(z+3)}=\frac{A}{z-1}+\frac{B}{z+3}=\frac{A(z+3)+B(z-1)}{(z-1)(z+3)} \qquad (1).$$

Substitute for $z=1$ into the left-hand and right-hand sides of (1) and obtain 1=4A, hence

$A=\frac{1}{4}$.

Now substitute for $z=-3$ into the left-hand and right-hand sides of (1) and obtain

-3=-4B, hence $B=\frac{3}{4}$.

So we got

$$\frac{z}{z^2+2z-3}=\frac{1}{4(z-1)}+\frac{3}{4(z+3)}.$$

Using linearity of the inverse transform of Laplace and the table of the inverse transform of Laplace we get:

$$L^{-1}\left\{\frac{1}{4(z-1)}+\frac{3}{4(z+3)}\right\}=\frac{1}{4}L^{-1}\left\{\frac{1}{z-1}\right\}+\frac{3}{4}L^{-1}\left\{\frac{1}{z+3}\right\}=\frac{1}{4}e^{t}+\frac{3}{4}e^{-3t}.$$

Answer: $\frac{1}{4}e^{t}+\frac{3}{4}e^{-3t}$.