Answer on Question #57025 – Math – Differential Equations

The response $x(t)$ of system to an input $u(t)$ is given by the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 2\frac{du}{dt} + 4u$$

Find the transfer function of the system and draw the pole-zero plot.

Solution

The differential equation:

$$\ddot{x} + 6\dot{x} + 13x = 2\dot{u} + 4u$$

The equation in the operator form using differential operator s:

$$(s^2 + 6s + 13)x = (2s + 4)u$$

Find the transfer function of the system:

$$W(s) = \frac{x}{u} = \frac{2s + 4}{s^2 + 6s + 13}$$

Pole: value of s that makes $T.F. \to \infty$

Pole: $s^2 + 6s + 13 = 0$

$$D = b^2 - 4ac = 6^2 - 4 \times 13 = 36 - 52 = -16$$

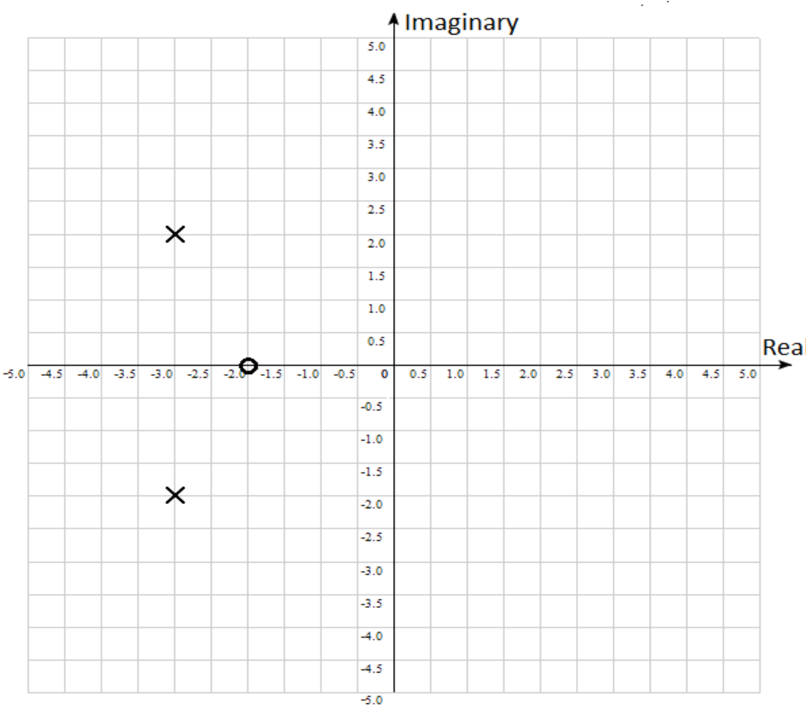
$$s = \frac{-b \pm \sqrt{D}}{2a} = \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2} = -3 \pm 2i$$

Zero: value of s that makes $T.F. \to 0$

Zero: $2s + 4 = 0$

$$s = -2$$

Pole-zero plot:



Answer: the transfer function of the system is $W(s) = \frac{2s+4}{s^2+6s+13}$, pole: $s = -3 + 2i$ and $s = -3 - 2i$, zero: $s = -2$.