

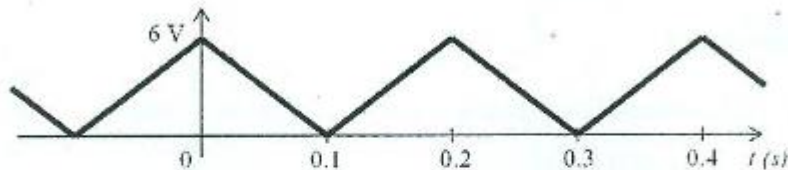
Answer on Question #57024 – Math – Calculus

please help me this question

http://i1376.photobucket.com/albums/ah26/john1x96/ex1_zpsmltfronf.png

Question 5 (20 marks)

The figure below shows a 'sawtooth' voltage signal $V(t)$. Find the Fourier series for $V(t)$.



Question 6

Find the Fourier transforms of the given functions:

(a) (10 marks)

$$f(t) = \begin{cases} 1 & |t| < 1 \\ 0 & |t| > 1. \end{cases} \quad \frac{1}{j\omega} (e^{-j\omega} - e^{j\omega})$$

(b) (10 marks)

$$g(t) = \begin{cases} \sin t & |t| \leq \pi \\ 0 & |t| > \pi. \end{cases}$$

Hint: You may wish to use the formula

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)].$$

Solution

$$5. f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n t}{L} + b_n \sin \frac{2\pi n t}{L} \right) \text{ where}$$

$$a_0 = \frac{1}{2L} \int_{-L}^L f(t) dt = \frac{2}{0.4} \int_0^{0.1} 60t dt = 300 \frac{t^2}{2} \Big|_{t=0}^{t=0.1} = \frac{3}{2};$$

$$a_n = \frac{1}{L} \int_{-L}^L f(t) \cos \left(\frac{2\pi n t}{L} \right) dt = \frac{2}{0.2} \int_0^{0.1} 60t \cos \left(\frac{2\pi n t}{0.2} \right) dt = 6 \frac{(-1)^{n-1}}{\pi^2 n^2}$$

$$b_n = \frac{1}{L} \int_{-L}^L f(t) \sin\left(\frac{2\pi nt}{L}\right) dt = \frac{2}{0.2} \int_0^{0.1} 60t \sin\left(\frac{2\pi nt}{0.2}\right) dt = 6 \frac{(-1)^n}{\pi n}$$

$$f(t) = \frac{3}{2} + 6 \sum_{n=1}^{\infty} \left(\frac{(-1)^{n-1}}{\pi^2 n^2} \cos \frac{2\pi nt}{L} + \frac{(-1)^n}{\pi n} \sin \frac{2\pi nt}{L} \right)$$

6.

$$(a) F(k) = \int_{-\infty}^{\infty} e^{-2\pi ikt} f(t) dt = \int_{-1}^1 e^{-2\pi ikt} dt = -\frac{1}{2\pi ik} e^{-2\pi ikt} \Big|_{t=-1}^{t=1} = \frac{\sin 2\pi k}{2\pi k}$$

$$(b) G(k) = \int_{-\infty}^{\infty} e^{-2\pi ikt} g(t) dt = \int_{-\pi}^{\pi} e^{-2\pi ikt} \sin t dt =$$

$$= \int_{-\pi}^{\pi} (\cos 2\pi kt - i \sin 2\pi kt) \sin t dt = \int_{-\pi}^{\pi} \cos 2\pi kts \sin t dt -$$

$$-i \int_{-\pi}^{\pi} \sin 2\pi kts \sin t dt = -\frac{i \sin 2\pi^2 k}{4\pi^2 k^2 - 1}$$