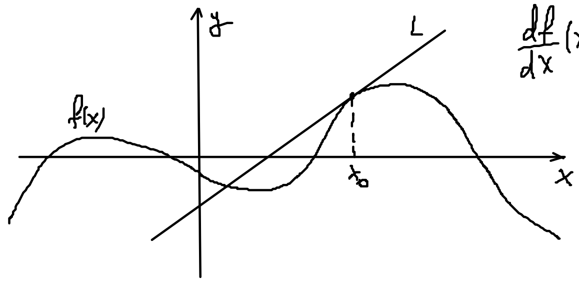
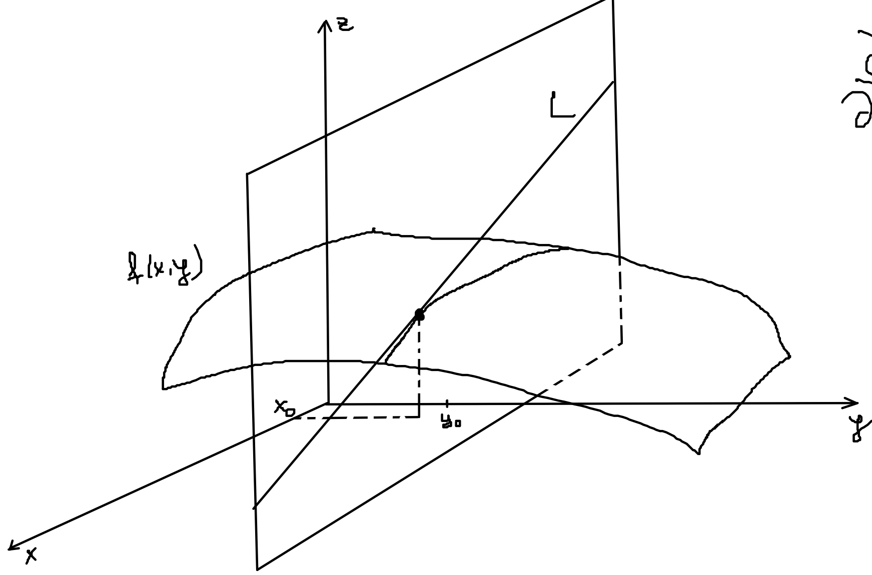
14.3 Geometric interpretation (of partial derivatives)

Given a function $f:\mathbb{R}^1 \to \mathbb{R}^1$ we know that the derivative $\frac{df}{dx}(x_0)$ gives us the slope of the tangent line to the graph of the function f at x_0.



$\frac{df}{dx}(x_0) = \text{slope of } L.$

If we have a function $f:\mathbb{R}^2 \to \mathbb{R}^1$ its graph is a surface in $\mathbb{R}^3$.

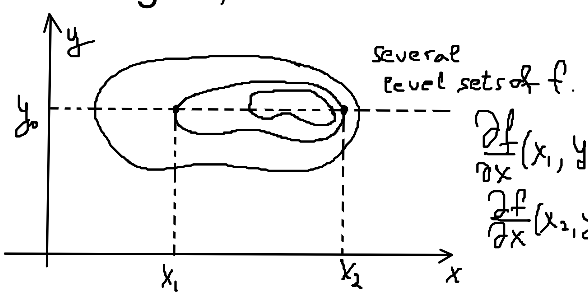


$\frac{\partial f}{\partial x}(x_0, y_0) = \text{slope of } L$

Thus, for a function $f:\mathbb{R}^2 \to \mathbb{R}^1$ the partial derivative $\frac{\partial f}{\partial x}(x_0, y_0)$ is the slope of the tangent line to the intersection of the graph of f with the plane $y = y_0$ at the point (x_0, y_0). Similarly $\frac{\partial f}{\partial y}(x_0, y_0)$ is the slope of the tangent line to the intersection of the graph of f with the plane $x = x_0$.

We can also investigate how partial derivatives relate to level sets.

Once again, we have $f:\mathbb{R}^2 \to \mathbb{R}^1$



$\frac{\partial f}{\partial x}(x_1, y_0)$ is small

$\frac{\partial f}{\partial x}(x_2, y_0)$ is large

$\frac{\partial f}{\partial x}(x, y_0)$ measures the rate of change of f with respect to x as we move along the line $y = y_0$