Answer on Question #57002 – Math – Geometry

Question

1. An isosceles trapezoid has an area of $A = 40\ \mathrm{m}^2$ and an altitude of $h = 2$ meters. Its two bases have a ratio of 2 is to 3. What are the lengths of the bases and one diagonal of the trapezoid?

Solution

Let the length of one base be $2x$ and the length of the other base be $3x$ then we obtain

$$A = \frac{3x + 2x}{2} h$$

$$x = \frac{2A}{5h} = \frac{2 \cdot 40\mathrm{m}^2}{5 \cdot 2\mathrm{m}} = 8\mathrm{m}$$

Thus the lengths of trapezoid bases are $16\mathrm{m}$ and $24\mathrm{m}$.

The altitude of this trapezoid divides the larger base into parts of length $20\mathrm{m}$ and $4\mathrm{m}$. the bigger of these parts together with the altitude and the diagonal form the right triangle (diagonal is the hypotenuse). Thus, according to the Pythagorean theorem the length of the diagonal is given by

$$d = \sqrt{(20\mathrm{m})^2 + (4\mathrm{m})^2} = 4\sqrt{26}\mathrm{m}$$

Answer: 24 m, 16 m, $4\sqrt{26}\mathrm{m}$

Question

2. A piece of wire of length 52 meters is cut into two parts. Each part is then bent to form a square. It is found that the combined area of the two squares is 109 m^2. Find the measure of the sides of the two squares.

Solution

Let the length of the one part be x, then the length of the other part will be $52 - x$. Side of the first square has length of $\frac{x}{4}$ and the side of the other one has the length of $\frac{52-x}{4}$. Thus the sum of their areas is $\left(\frac{x}{4}\right)^2 + \left(\frac{52-x}{4}\right)^2$. It's given that

$$\left(\frac{x}{4}\right)^2 + \left(\frac{52 - x}{4}\right)^2 = 109$$

$$x^2 + (52 - x)^2 = 109 \cdot 4^2$$

$$x^2 + 2704 - 104x + x^2 = 1744$$

$$x^2 - 52x + 480 = 0$$

$$(x - 12)(x - 40) = 0$$

This equation has two solutions: $x = 12\ \mathrm{m}$ and $x = 40\ \mathrm{m}$. They represent the lengths of the two parts of the wire. Thus the sides of the squares are: $\frac{12\ \mathrm{m}}{4} = 3\ \mathrm{m}$ and $\frac{40\ \mathrm{m}}{4} = 10\ \mathrm{m}$.

Answer: 3m, 10m.