

**Question #56912, Math / Calculus**

please help me this question

[http://i284.photobucket.com/albums/ll8/tan\\_pham1/cal\\_zpsljiej1vb.png](http://i284.photobucket.com/albums/ll8/tan_pham1/cal_zpsljiej1vb.png)

Integrate term by term the Fourier series expansion obtained in Example square wave

$$f(t) = \begin{cases} -1 & (-\pi < t < 0) \\ 1 & (0 < t < \pi) \end{cases}$$

$$f(t + 2\pi) = f(t)$$

**Answer.**

$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$  where

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt = \frac{1}{2\pi} \left[ \int_{-\pi}^0 (-1) dt + \int_0^{\pi} dt \right] = -\pi + \pi = 0;$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt dt = \frac{1}{\pi} \left[ - \int_{-\pi}^0 \cos nt dt + \int_0^{\pi} \cos nt dt \right] = \\ &= \frac{1}{\pi} \left[ -\frac{1}{n} \sin n\pi + \frac{1}{n} \sin n\pi \right] = 0; \end{aligned}$$

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt dt = \frac{1}{\pi} \left[ - \int_{-\pi}^0 \sin nt dt + \int_0^{\pi} \sin nt dt \right] = \\ &= \frac{2}{\pi n} (1 - \cos n\pi) = \frac{2(1 - (-1)^n)}{\pi n}. \end{aligned}$$

So  $f(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n} \sin nt.$