Answer on Question #56909 – Math – Calculus

Question

State the domain and range of the function. Show work.

$$k(x) = \frac{4x+3}{x^2-1}$$

Solution

Domain of function

Here we have only one restriction on values of x: denominator cannot be equal to zero:

$$x^2 - 1 \neq 0,$$

$$x \neq 1 \text{ or } x \neq -1.$$

Hence domain is $(-\infty,-1) \cup (-1,1) \cup (1,+\infty)$.

Range of function

Here we have only one restriction on values of k: there exists a value of x such that $k = \frac{4x+3}{x^2-1}$.

Rewrite the last equality in the following form:

$\frac{4x+3}{x^2-1} - k = \frac{4x+3-k(x^2-1)}{x^2-1} = \frac{-kx^2+4x+(3+k)}{x^2-1} = 0$, then

$$-kx^2 + 4x + (3+k) = 0$$

is a quadratic equation for $k \neq 0$,

its discriminant is

$D = 16 - 4 \cdot (-k) \cdot (3+k) = 16 + 4k(k+3) = 4k^2 + 12k + 16 = 4(k^2 + 3k + 4) =$

$= 4\left(k^2 + 2 \cdot \frac{3}{2}k + \frac{9}{4} + 4 - \frac{9}{4}\right) = 4\left(\left(k^2 + 2 \cdot \frac{3}{2}k + \frac{9}{4}\right) + \frac{4 \cdot 4 - 9}{4}\right) = 4\left(\left(k + \frac{3}{2}\right)^2 + \frac{7}{4}\right) > 0$, hence the quadratic equation always has a root for any value of $k \neq 0$.

If $k = 0$, then $4x + 3 = 0$, hence $x = -\frac{3}{4}$. So there exists a value of x such that $k = 0$.

Thus, k can take any real values, hence the range of function $k(x) = \frac{4x+3}{x^2-1}$ is $(-\infty;\ +\infty)$.

Answer: $(-\infty,-1) \cup (-1,1) \cup (1,+\infty)$; $(-\infty;\ +\infty)$.