Question #56905, Math / Calculus

1: If you deposit $1,000 into an account that pays 4% interest compounded continuously, how long will it take the account to grow to $2,000? Show work. answer 17.3 years.

2: If you pour a cup of coffee that is 200degrees F, and set it on a desk in a room that is 68Degrees F, and 10 minutes later it is 145degrees F, what temperature will it be 15 minutes after you originally poured it? Show work. answer 126.8 Degrees F

3: The radioactive substance krypton-91 has a very short half-life of about 10 seconds. What is the decay constant for this substance? Show work. answer 0.0693.

Solution:

1. We have a standard continuously compounded interests formula:

$$A_t = A_0 e^{-rt}$$

In our problem we have:

$$A_0 = 1000, \qquad A_t = 2000, \qquad r = 0.04\ (4\%)$$

We need to find t

$$\frac{A_t}{A_0} = e^{-rt}$$

$$\ln\left(\frac{A_t}{A_0}\right) = -rt$$

$$t = \frac{\ln\left(\frac{A_t}{A_0}\right)}{r} = \frac{\ln\left(\frac{2000}{1000}\right)}{0.04} = \frac{\ln(2)}{0.04} = 17.3$$

2. To calculate heat flow we have to solve the heat equation. in practice we tend to use simple approximations. In everyday life heat flow tends to be well described by Newton's equation:

$$\frac{dQ}{dt} \propto \Delta T$$

So the greater the temperatures difference the faster the heat flow. The temperature difference as a function of time will look like:

$$T(t) = T_0 e^{-\alpha t}$$

where T_0 is the initial temperature and α is a constant related to the thermal conductivity (large α means high thermal conductivity).
In our case, we can write:

$$145 = 200e^{-\alpha*10}$$

From this we will find α

$$\frac{145}{200} = e^{-\alpha*10}$$

$$0.725 = e^{-\alpha*10}$$

$$\ln(0.725) = -\alpha * 10$$

$$\alpha = -\frac{\ln(0.725)}{10} \approx 0.03$$

Now we can find the temperature in 15 minutes:

$$T(15) = 200e^{-0.03*15} \approx 126.8$$

3. We have a simply formula of half-life time:

$$t_{1/2} = \frac{\ln(2)}{\lambda}$$

where λ is the exponential decay constant. We have that for krypton-91, $t_{1/2} = 10$ seconds. So we can easily find λ

$$\lambda = \frac{\ln(2)}{t_{1/2}} = \frac{0.69314}{10} = 0.0693$$