Answer on Question #56888 – Math – Calculus

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} =$$

Solution

It is well-known that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \zeta(2) = \frac{\pi^2}{6},$$

where $\zeta(s)$ is the Riemann zeta function of a complex variable $s = a + i \cdot b$.

Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^2} - \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots\right) =$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{2^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right) = \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{3}{4}\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{3}{4} \cdot \frac{\pi^2}{6} = \frac{\pi^2}{8}.$$

Answer: $\frac{\pi^2}{8}$.
