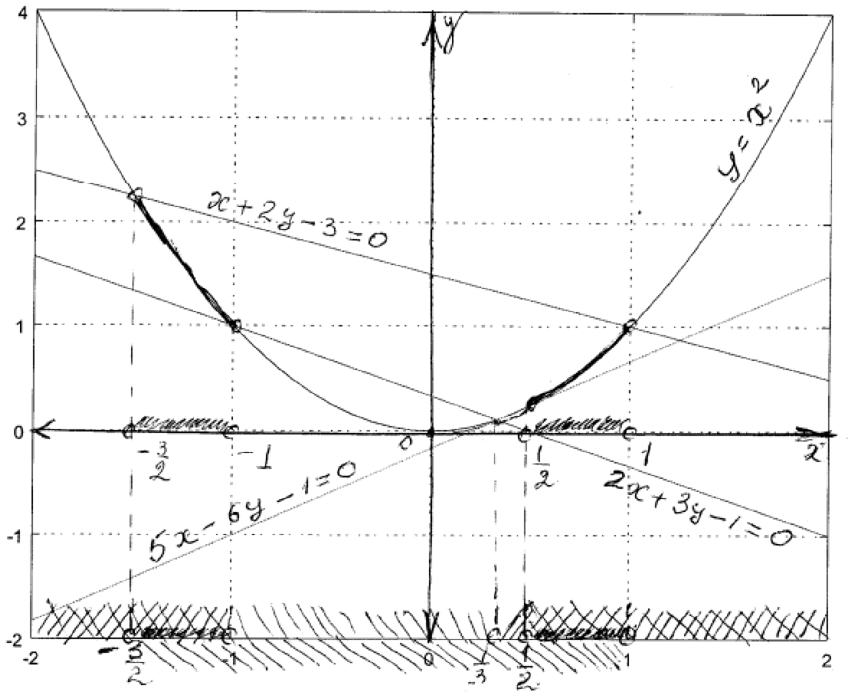
Problem.

Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines $2x + 3y - 1 = 0$; $x + 2y - 3 = 0$; $5x - 6y - 1 = 0$.

Solution

The given triangle is defined by the system of inequalities (see Figure)

$$\begin{cases} 2x+3y-1 \ge 0 \\ x+2y-3 \le 0 \\ 5x-6y-1 \le 0 \end{cases}$$



The point (t,t^2) will be the inner point of this triangle when

$$\begin{cases} 2t+3t^2-1>0 \\ t+2t^2-3<0 \\ 5t-6t^2-1<0 \end{cases}$$

$$\begin{cases} (t+1)(t-\frac{1}{3})>0 \\ (t+\frac{3}{2})(t-1)<0 \\ (t-\frac{1}{3})(t-\frac{1}{2})>0 \end{cases}$$

We will find a solution to this system of inequalities by intervals (see Figure)

$$t \in (-\frac{3}{2},-1) \cup (\frac{1}{2},1)$$

Answer. The point (α, α^2) lies inside the given triangle when α belongs the set $(-\frac{3}{2},-1) \cup (\frac{1}{2},1)$.