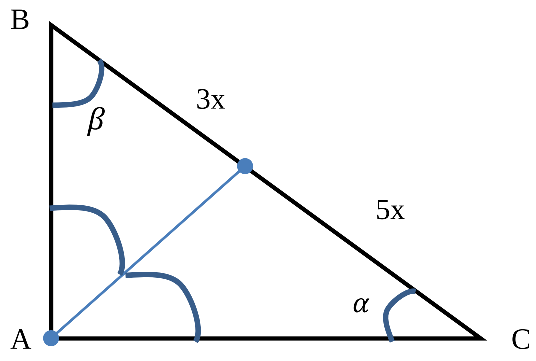
Answer on Question #56862 – Math – Geometry

1. In a right triangle, the bisector of the right angle divides the hypotenuse in the ratio of 3:5. Determine the measures of the acute angles of the triangle.

Solution



From properties of bisector, we know that:

$$\frac{AB}{AC} = \frac{3x}{5x} = \frac{3}{5}$$

Also we know:

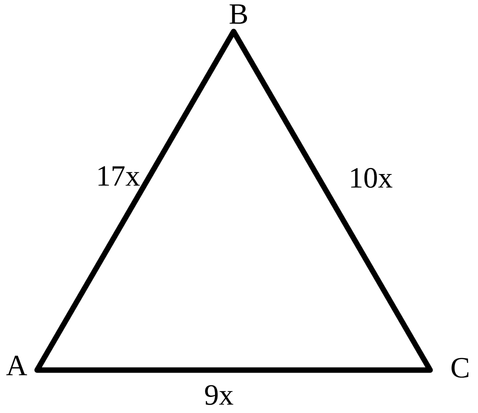
$$\frac{AB}{AC} = \tan\alpha = \frac{3}{5} \quad => \quad \alpha = \arctan\frac{3}{5}$$

$$\frac{AB}{AC} = \cot\beta = \frac{3}{5} \quad => \quad \beta = \operatorname{arccot}\frac{3}{5}$$

Answer: $\arctan\frac{3}{5}$, $\operatorname{arccot}\frac{3}{5}$.

2. The lengths of the sides of a triangle are in the ratio of 17:10:9. Find the lengths of the three sides if the area of the triangle is 576cm^2.

Solution



Geron's formula of area is

$$S = \sqrt{p(p-a)(p-b)(p-c)}, \;\; p = \frac{a+b+c}{2}$$

So we have:

$$p = \frac{17x + 10x + 9x}{2} = 18x$$

$$S = 576 = \sqrt{18x(18x-17x)(18x-10x)(18x-9x)} = \sqrt{18x(x)(8x)(9x)}$$
$$= \sqrt{1296x^4} = 36x^2$$

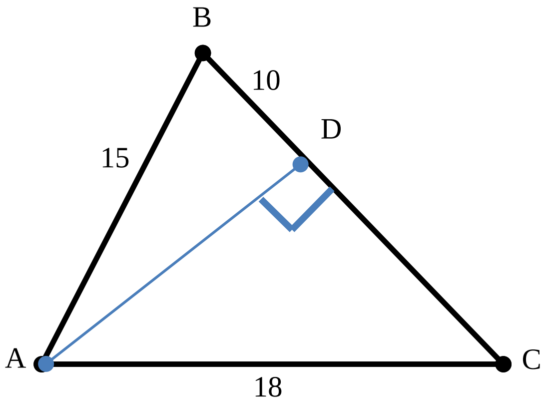
We can find x:

$$x = \sqrt{\frac{S}{36}} = \sqrt{\frac{576}{36}} = 4$$

So the sides of triangle are
$AB = 17x = 68, BC = 10x = 40, AC = 9x = 36.$
Answer: $68, 40, 36.$

3. In an acute triangle ABC, an altitude AD is drawn. Find the area of triangle ABC if AB = 15 inches, AC = 18 inches and BD = 10 inches.

<u>Solution</u>



By Pythagorean Theorem,

$$AD = \sqrt{AB^2 - BD^2} = \sqrt{225 - 100} = \sqrt{125} = 5\sqrt{5}$$

$$DC = \sqrt{AC^2 - AD^2} = \sqrt{324 - 125} = \sqrt{199}$$

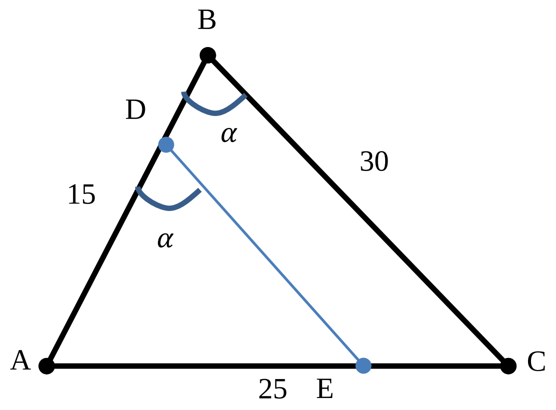
Next, $BC = BD + DC = 10 + \sqrt{199}$.
Finally we can calculate the area of this triangle:

$$S = \frac{1}{2} BC \cdot AD = \frac{1}{2} \cdot \left(10 + \sqrt{199}\right) \cdot 5\sqrt{5} \approx 61.5$$

Answer: 61.5 inches^2.

4. Given triangle ABC whose sides are AB = 15 inches, AC = 25 inches and BC = 30 inches. From a point D on side AB, a line DE is drawn to a point E on side AC such that angle ADE is equal to angle ABC. If the perimeter of triangle ADE is 28 inches. Find the lengths of line segments BD and CE.

Solution



Triangle ABC is similar to triangle ADE (because they have same angles: given $\angle ABC = \angle ADE = \angle B = \alpha$, by construction $\angle BAC = \angle DAE = \angle A$). Then

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

Substitute for known values of sides

$$\frac{AD}{15} = \frac{AE}{25} = \frac{DE}{30}$$

It is given that

$$P = 28 = AD + AE + DE$$

We can form a system of equations

$$\begin{cases} 28 = AD + AE + DE \\ \dfrac{AD}{15} = \dfrac{AE}{25} \\ \dfrac{AE}{25} = \dfrac{DE}{30} \end{cases}$$

$$\begin{cases} 28 = \dfrac{3}{5}AE + AE + \dfrac{6}{5}AE \\ AD = \dfrac{15 * AE}{25} = \dfrac{3}{5}AE \\ DE = \dfrac{30 * AE}{25} = \dfrac{6}{5}AE \end{cases}$$

$$28 = \frac{14}{5} AE \quad => AE = 10 \qquad =>$$

$$\begin{cases} AE = 10 \\ AD = \frac{3}{5} \cdot 10 \\ DE = \frac{6}{5} \cdot 10 \end{cases}$$

$$\begin{cases} AE = 10 \\ AD = 6 \\ DE = 12 \end{cases}$$

So we can easily find

$$BD = AB - AD = 15 - 6 = 9$$

$$CE = AC - AE = 25 - 10 = 15$$

Answer: $BD = 9$, $CE = 15$.