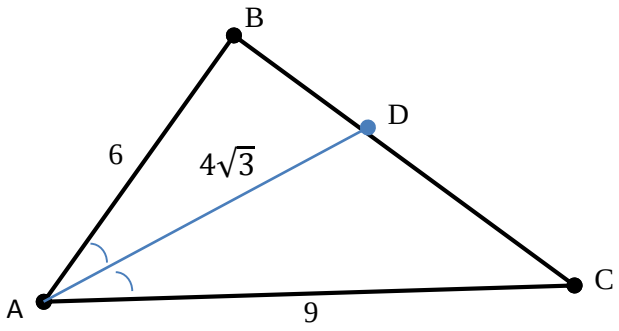
Answer on Question #56861 – Math - Geometry

1. Find the area of a triangle if its two side measure 6 inches and 9 inches, and the bisector of the angle between the sides is 4 square root of 3 inches.
2. In a right triangle, a line perpendicular to the hypotenuse drawn from the midpoint of one of the sides divides the hypotenuse into segments which are 10 cm and 6 cm long. Find the lengths of the two sides of the triangle.
3. The base of an isosceles triangle and the altitude drawn from one of the congruent sides are equal to 18 cm and 15 cm, respectively. Find the lengths of the sides of the triangle.

Solution

1.



From Stuart's theorem and angle bisector theorem we can find the bisector measure

$$AD = \frac{\sqrt{AB * AC * (AB + AC + BC) * (AB + AC - BC)}}{AB + AC}$$

From this formula we can find BC:

$$AD^2 = \frac{AB * AC * (AB + AC + BC) * (AB + AC - BC)}{(AB + AC)^2}$$

$$AD^2 = \frac{AB * AC * ((AB + AC)^2 - BC^2)}{(AB + AC)^2}$$

$$\frac{AD^2 * (AB + AC)^2}{AB * AC} = (AB + AC)^2 - BC^2$$

$$BC = \sqrt{(AB + AC)^2 - \frac{AD^2 * (AB + AC)^2}{AB * AC}}$$

$$BC = \sqrt{(6 + 9)^2 - \frac{4\sqrt{3}^2 * (6 + 9)^2}{6 * 9}}$$

$$BC = \sqrt{225 - \frac{48 * 225}{54}} = \sqrt{225 - 200} = 5$$

We can find the area of triangle by Geron's formula:

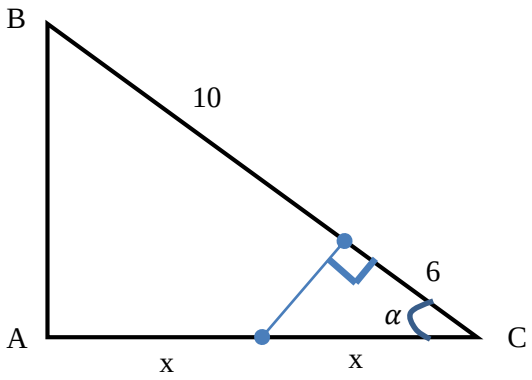
$$S = \sqrt{p(p-a)(p-b)(p-c)}, \quad p = \frac{a+b+c}{2}$$

So,

$$p = \frac{6+9+5}{2} = 10$$

$$S = \sqrt{10 * (10-6) * (10-9) * (10-5)} = \sqrt{10 * 4 * 1 * 5} = \sqrt{200} = 10\sqrt{2}\ inches$$

2.



We can write a system of equations:

$$\begin{cases} \cos\alpha = \frac{x+x}{10+6} = \frac{2x}{16} \\ \cos\alpha = \frac{6}{x} \end{cases}$$

From this we can say:

$$\frac{2x}{16} = \frac{6}{x}$$

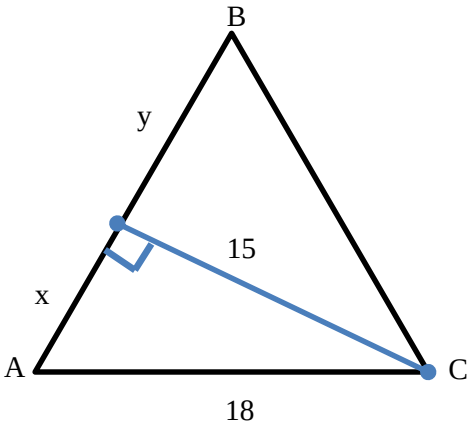
$$2x^2 = 96$$

$$x^2 = 48 \quad => \quad x = 4\sqrt{3}$$

Then AC = $2x = 8\sqrt{3}$, and using Pythagorean theorem get

AB =$\sqrt{BC^2 - AC^2} = \sqrt{16^2 - 8\sqrt{3}^2} = \sqrt{256 - 192} = \sqrt{64} = 8.$

3.



First step, we need to find x by Pythagorean Theorem:

$$x = \sqrt{18^2 - 15^2} = \sqrt{324 - 225} = 3\sqrt{11}$$

Now we will apply Pythagorean Theorem to the triangle which contains y and altitude:

$$(x + y)^2 = 15^2 + y^2$$

$$\left(3\sqrt{11} + y\right)^2 = 15^2 + y^2$$

$$99 + 6\sqrt{11}y + y^2 = 225 + y^2$$

$$6\sqrt{11}y = 225 - 99$$

$$y = \frac{126}{6\sqrt{11}} = \frac{21}{\sqrt{11}} = \frac{21\sqrt{11}}{11}$$

So the lengths of sides AB and BC of triangle are

$AB = BC = x + y = 3\sqrt{11} + \frac{21\sqrt{11}}{11} = \frac{54\sqrt{11}}{11}$.