Answer on Question #56705 – Math – Analytic Geometry

Question

Trisect the segment connecting P(-5,2) and Q(3,-4).

Solution

Segment PQ, that connects points P(-5,2) and Q(3,4), is divided into three equal parts by points A(x_a, y_a), B(x_b, y_b).

Besides, $x_p < x_a < x_q$, $x_p < x_b < x_q$, $y_q < y_a < y_p$, $y_q < y_b < y_p$, that is, $-5 < x_a < 3$, $-5 < x_b < 3$, $-4 < y_a < 2$, $-4 < y_b < 2$.

Points P, A, B, Q lie on the one line, so

$\frac{x_a - x_p}{y_a - y_p} = \frac{x_b - x_p}{y_b - y_p} = \frac{x_q - x_p}{y_q - y_p} = \frac{8}{-6} = -\frac{4}{3}$

Length of original segment $L_{PQ} = \sqrt{(x_q - x_p)^2 + (y_q - y_p)^2} = \sqrt{8^2 + 6^2} =$
$= 10$,

Length of $L_{PA} = \frac{1}{3} L_{PQ} = \frac{10}{3} = \sqrt{(x_a - x_p)^2 + (y_a - y_p)^2} =$
$\sqrt{(\frac{4^2}{3^2} + 1)(y_a - y_p)^2} = \pm\frac{5}{3}(y_a - y_p)$, we obtain $y_a - y_p = 2$ or $y_a - y_p = -2$, but the case of $y_a - y_p = 2$ is impossible, because $y_a - y_p < 0$. Then $y_a - y_p = -2$, hence $y_a = y_p - 2 = 2 - 2 = 0$.

It follows from $\frac{x_a - x_p}{y_a - y_p} = -\frac{4}{3}$ and $y_a - y_p = -2$ that

$x_a - x_p = -\frac{4}{3}(-2) = \frac{8}{3}$, hence

$x_a = \frac{8}{3} + x_p = \frac{8}{3} - 5 = -\frac{7}{3}$.

Similarly for

$L_{PB} = \frac{2}{3} L_{PQ} = \frac{20}{3} = \sqrt{(x_b - x_p)^2 + (y_b - y_p)^2} = \sqrt{(\frac{4^2}{3^2} + 1)(y_b - y_p)^2} =$
$\pm\frac{5}{3}(y_b - y_p)$, we obtain that $y_b - y_p = 4$ or $y_b - y_p = -4$, but the case of

$y_b - y_p = 4$ is impossible, because $y_b - y_p < 0$.

Then $y_b - y_p = -4$, hence $y_b = y_p - 4 = 2 - 4 = -2$.

It follows from $\frac{x_b - x_p}{y_b - y_p} = -\frac{4}{3}$ and $y_a - y_p = -2$ that

$x_b - x_p = -\frac{4}{3}(-4) = \frac{16}{3}$, hence

$x_b = \frac{16}{3} + x_p = \frac{16}{3} - 5 = \frac{1}{3}$. Thus, $(x_a, y_a)=\left(\frac{-7}{3}, 0\right)$, $(x_b, y_b)=\left(\frac{1}{3}, -2\right)$.

Answer: the points that divided PQ into three equal parts are A($-\frac{7}{3}$,0) and B($\frac{1}{3}$, -2).