Answer on Question #56700 – Math– Combinatorics | Number Theory

23. Consider $xyz = 24$, where $x, y, z \in I$, then
(a) Total number of positive integral solutions for x, y, z are 81
(b) Total number of positive integral solutions for x, y, z are 90
(c) Total number of positive integral solutions for x, y, z are 30
(d) Total number of positive integral solutions for x, y, z are 120

Solution

Because $24 = 2^3 \cdot 3^1$, then there are 4 prime positive divisors of 24. It follows, that total number of positive integer solutions for x, y, z is $\overline{A_3^4} = 3^4 = 81$. The total number of all integer solutions for x, y, z is $81 \cdot 2^3 = 81 \cdot 8 = 648$.

Answer: (a).

24: If ${}^{n}C_{r+1} = (m^2 - 8) \cdot {}^{n-1}C_r$; then possible value of $'m'$ can be
(a) 4 (b) 2 (c) 3 (d) -5

Solution

We have:
${}^{n}C_{r+1} = \frac{n!}{(n-r-1)!(r+1)!} = \frac{n\cdot(n-1)!}{(r+1)\cdot(n-r-1)!r!} = (m^2-8)\cdot {}^{n-1}C_r = (m^2-8)\cdot\frac{(n-1)!}{(n-r-1)!r!}$,
hence

$$n = (m^2 - 8)(r + 1)$$

and $\frac{n}{r+1}$ must be a positive integer, $1 < \frac{n}{r+1} \le n$, hence $m^2 - 8 > 1$, that is, $m^2 > 9$.

Thus, $m \ne 2$, $m \ne 3$, and possible value of $'m'$ can be **4** (for $n = 16, r = 1$, for example) or **(-5)** (for $n = 17, r = 0$, for example).

Answer: (a) or (d).
