Answer on Question #56699 - Math - Combinatorics | Number Theory

21. Total number of four letters words that can be formed from the letters word 'DPSRKPURAM', is given by:
(a) ${}^{10}C_4.(4!)$ (b) 2190 (c) Coefficient of x^4 in $\{4!.(1+x+x^2)(1+x)^6\}$
(d) Coefficient of x^4 in $\{3!.(1+x)^6(1+(x+1)^2)^2\}$.

Solution

Case 1: all four letters distinct. There are 8 different letters in our long word. We can choose 4 distinct letters in $\binom{8}{4}$ ways, and for each way arrange them in 4! ways, for a total of $\binom{8}{4}4!$.

Case 2: two P's , the rest distinct. We can choose the places where the P's go in $\binom{4}{2}$ ways. For each such choice, there are (7)(6) ways to fill the remaining slots with two distinct letters chosen from the remaining 7.

Case 3: two R's , the rest distinct. The same analysis, answer as in Case 2.

Case 4: Two P's, two R's. The places, where the P's go, can be chosen in $\binom{4}{2}$ ways.

Total number to form four letter words:

$\binom{8}{4}4! + 2\cdot\binom{4}{2}(7)(6) + \binom{4}{2} = \frac{8!4!}{4!4!} + \frac{84\cdot 4!}{2!2!} + \frac{4!}{2!2!} = 8\cdot7\cdot6\cdot5 + 84\cdot6 + 6 = 2190$

Notice that $3!.(1+x)^6(1+(x+1)^2)^2 = 6x^{10} + 60x^9 + 282x^8 + 816x^7 + 1602x^6 + 2220x^5 + 2190x^4 + 1512x^3 + 696x^2 + 192x + 24$

Answer: (d) Coefficient of x^4 in $\{3!\,(1+x)^6(1+(x+1)^2)^2\}$.

22. Consider seven digit number $x_1x_2x_3x_4 \dots x_7$, where $x_1, x_2, \dots, x_7 \neq 0$, having the property that x_4 is the greatest digit and digits towards the left and right of x_4 are in decreasing order, then total number of such numbers in which all digits are distinct is given by:
(a) ${}^9C_7.{}^6C_3$ (b) ${}^9C_2.{}^6C_4$ (c) $3.{}^9C_7.{}^5C_1$ (d) $2.{}^9C_2.{}^5C_2$.

Solution

There are 9 different numbers which we can use. We can choose 7 distinct numbers in ${}^9C_7 = \binom{9}{7}$ ways. One of them will be the greatest, so we must divide rest 6 digits for two groups. We can choose 3 digits from 6 in ${}^6C_3 = \binom{6}{3}$ ways. By multiplication principle, the total number is ${}^9C_7.{}^6C_3 = \binom{9}{7}\binom{6}{3}$.

Answer: (a) $\binom{9}{7}\binom{6}{3}$.